

# Numerical stability of explicit and implicit co-simulation methods

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## ABSTRACT

To couple different solvers in the framework of a multidisciplinary analysis or in order to parallelize the dynamic analysis of a monolithic system, co-simulation methods may be applied advantageously, see e.g.[1, 2]. Within a co-simulation approach, the subsystems are integrated by specific solvers; data exchange is accomplished only at certain user-defined macro-time points. Therefore, it is possible to apply different optimized subsystem integrators, which may greatly increase the computational efficiency of the overall simulation. Due to the approximation of the coupling variables by polynomials and as a result of the data exchange between the subsystems, errors are introduced, which may entail severe stability problems. Hence, the development of stabilized coupling techniques is of special interest.

Concerning the coupling algorithms in mechanical systems, usually two methods can be distinguished, namely applied force coupling [4] and algebraic constraints coupling [3]. Here, we only consider the case based on applied force coupling, which can further be divided into force/force-, force/displacement- and displacement/displacement coupling approaches. Moreover, we have to distinguish between explicit and implicit algorithms. Applying an implicit approach the coupling variables are calculated in an iterative way, whereas the coupling variables are evaluated only once in connection with explicit algorithms. The numerical stability behaviour of both, explicit and implicit algorithms will be discussed here.

The stability of time integration schemes is defined by Dahlquist's test equation. From the mechanical point of view, this equation can be interpreted as the complex representation of the equations of motion of the autonomous linear mass-spring-damper oscillator. To analyse the stability of co-simulation approaches, we consider two coupled Dahlquist's equations so that the conventional linear stability analysis can be applied. Consequently, the stability of the co-simulation method can be determined by calculating the spectral radius of the governing system of recurrence equations. The numerical stability of classical explicit and implicit co-simulation techniques is investigated here. Also, modified coupling approaches are discussed, which show an improved stability behaviour.

## REFERENCES

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