Thermodynamically consistent integration of thermoelastic solids using a temperature-based GENERIC framework

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Abstract

This work deals with the thermodynamically consistent (TC) time integration of thermoelastic systems with polyconvex energy density functions using the notion of the tensor-cross-product. While energy-momentum preserving integrators are well-known for conservative (isothermal) mechanical systems, Romero introduced in [2, 3] the new class of TC integrators.
While [3] dealt with the sample application of thermo-elastodynamics, the scope of application was extended in [6] to coupled thermo-viscoelastodynamics in temperature form. A first step towards the systematic design of a TC integrator is to cast the evolution equations into the GENERIC (General Equation for Non-Equilibrium Reversible-Irreversible Coupling) framework [1] which reveals additional underlying physical structures of the system. Relying on a polyconvex energy density function and using the notion of the tensor-cross-product [7] we arrive at a polyconvex version of the GENERIC framework. Further applying the notion of a discrete gradient leads to a TC integrator.
Using the entropy as the thermodynamical state variable as in [3, 4] the GENERIC framework possesses an easy structure. However, this choice of thermodynamical state variable only allows to prescribe entropy Dirichlet boundary conditions directly. This drawback can be compensated by using Lagrange-multipliers to be able to handle temperature Dirichlet boundary conditions leading to an extended system of algebraic equations to be solved, see [4]. Alternatively, the present work uses the temperature as the thermodynamical state variable, see also [5, 6]. The presentation includes several simulations with different boundary conditions and an energy-based termination criterion.

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