A Direct Solution Approach for Multi Timescale Optimal Control Problems

Matthias Bittner*, Benedikt Grüter†, Johannes Diepolder† and Florian Holzapfel†

*† Institute of Flight System Dynamics (FSD)
Technical University of Munich (TUM)
Boltzmannstraße 15, 85748 Garching, Germany
e-mail: m.bittner@tum.de, benedikt.grueter@tum.de,
johannes.diepolder@tum.de, florian.holzapfel@tum.de
web page: http://www.fsd.mw.tum.de/

ABSTRACT

One of the issues appearing in nonlinear optimal control problems with high fidelity simulation models are different time scales inherent to the model, resulting in *stiff differential equations*. Especially in mechanical systems, in many cases, fast dynamics can be recognized as *internal* in the event chain (small scale) dynamics, while slower dynamics often represent the more visible (large scale) *outer* part of the motion of a body. If the model may not be simplified by assuming the fast dynamics to be decayed instantly, the selection of an appropriate discretization grid is driven by the fast (yet small scale) dynamics that hardly influence the overall result. Consequently, a very fine discretization grid or higher order methods are required for all states in the problem, making the solution computationally expensive. In most high fidelity optimal control problems, the required fine grid adds more complexity than the additional states.

Several approaches have been suggested to overcome this issue, like Multi Rate Runge Kutta methods [1], a multi timescale collocation method [2] and the combination of single and multiple shooting [3]. In this work, a novel discretization scheme is proposed that combines the benefits of *Direct Collocation* with those of *Direct Multiple Shooting* by coupling the methods together.

The basic idea proposed here is to split the state vector and the dynamic system into fast and slow parts. The fast dynamics are discretized using a *Multiple Shooting* approach and the slow ones using a collocation scheme that is based on the same grid as the multiple shooting *segment nodes*. The number of Multiple Shooting integration grid points in between these segment nodes may be chosen freely – forming another, finer grid. In order to be able to evaluate the dynamic equations within the multiple shooting segments, an approximation of the slow states is required, that can be calculated using different interpolation algorithms.

The presented method is applied to an aircraft trajectory optimization problem, where a nonlinear high fidelity rigid body simulation model of an aerobatic aircraft is used that features very fast rotational dynamics compared to the translational motion. In the example a time optimal trajectory through a given race course respecting velocity, load factor and other constraints is calculated. The results show a significant reduction in the size of the numerical problem to be solved, also leading to a reduced calculation time.

REFERENCES

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