Explicit Partitioned Methods for Coupled Problems Based on Lagrange Multipliers

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ABSTRACT

In this talk we discuss the use of Lagrange multiplier techniques for the development of explicit partitioned methods for problems with interfaces. As a rule, a straightforward Lagrange multiplier formulation of such coupled problems enforces the continuity of the states across the interface and leads to monolithic discrete equations that are index 2 DAEs. These DAEs require careful time integration and may not be compatible with explicit time integration; see [1]. Consequently, the resulting monolithic problems do not lend themselves naturally to an explicit partitioned treatment. These issues can be partially overcome by referencing the Lagrange multiplier one time step ahead [1], [2], however such "forward increment multiplier" methods are still not purely explicit.

To obtain a Lagrange multiplier formulation that is fully compatible with explicit time integration we consider an alternative coupling condition which enforces the continuity of the time derivatives of the states across the interface. Assuming that the initial data are continuous across the interface, this alternative coupling condition implies the original one and yields an equivalent coupled problem. Application of Lagrange multiplier techniques to this problem leads to an index 1 DAE, which allows for a truly explicit treatment of the Lagrange multiplier.

We use this reduced index DAE system to generate two families of explicit partitioned methods that differ in the manner in which they treat the discrete Lagrange multiplier. The first family of methods eliminates this variable, leading to an expression for the "fluxes" exchanged between the subdomains in terms of the masses and forces at the interface nodes. The second family of methods uses a more traditional hybridization-like approach where the Lagrange multiplier is solved for explicitly and used as a Neumann boundary condition for a partitioned method.

To illustrate our approach we specialize the two families of explicit partitioned schemes to coupled elastodynamics problems and scalar advection-diffusion equations and present a series of numerical tests on subdomains with non-matching interface grids. These test confirm that the methods pass a patch test and exhibit second-order convergence.

REFERENCES

[1] Nicholas J. Carpenter, Robert L. Taylor, and Michael G. Katona. Lagrange constraints for transient finite element surface contact. *International Journal for Numerical Methods in Engineering*, 32(1):103–128, 1991.

[2] Michael A. Puso. A 3d mortar method for solid mechanics. *International Journal for Numerical Methods in Engineering*, 59(3):315–336, 2004.

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