Scalable High-Order Finite Element Discretizations and Solvers

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ABSTRACT

High-order finite element discretizations are a natural fit for future HPC hardware, because their order can be used to tune the performance, by increasing the FLOPs/bytes ratio, or to adjust the algorithm for different hardware. In this talk we present our work on scalable high-order finite element software that combines the modular finite element library MFEM [1], the *hypre* library of linear solvers [4], and the high-order shock hydrodynamics code BLAST [2,3]. We discuss the finite element abstractions provided by MFEM, which include arbitrary high-order H_1-conforming, discontinuous (L_2), H(div)-conforming, H(curl)-conforming and NURBS elements, defined on general high-order meshes. This compatible sequence of spaces (the de Rham complex) plays a critical role in discretization of coupled multi-physics problems, where different physics components naturally belong to different spaces in the sequence. For example, H_1 finite elements are used to discretize kinematic quantities (e.g. velocity, position), H(curl) finite elements are employed for the electric field in magneto-hydrodynamics (MHD) models, H(div) finite elements are used for the flux in radiation diffusion, and discontinuous L_2 finite elements represent thermodynamic quantities (e.g. internal energy). We explain how the MPI-based version of MFEM uses data structures and kernels from the *hypre* library to enable scalable finite element assembly in parallel. We further describe our approach for efficient implementation of high-order finite element assembly and evaluation in MFEM and its application to the evaluation of force matrices in the BLAST code, where we will also demonstrate the benefits of this approach with respect to strong scaling and GPU acceleration. Finally, we consider general non-conforming high-order adaptive refinement in MFEM with applications to coupled compressible radiation-hydrodynamics in BLAST and computational electromagnetic problems. Some preliminary work for efficient error estimation and scalable linear solvers in these settings will also be discussed.

REFERENCES

[1] MFEM: Modular finite element library, `http://mfem.org`

[2] BLAST: High-order finite element Lagrangian hydrocode, `http://www.llnl.gov/CASC/blast`

[3] V. A. Dobrev, T. V. Kolev, and R. N. Rieben, High-order curvilinear finite element methods for Lagrangian hydrodynamics, SIAM J. Sci. Comp., 34 (2012), pp. B606–B641.

[4] *hypre:* Scalable linear solvers, `http://www.llnl.gov/CASC/hypre`

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