A Mixed-Method B-Field Finite-Element Formulation for Incompressible, Resistive Magnetohydrodynamics

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ABSTRACT

Magnetohydrodynamics (MHD) models describe a wide range of plasma physics applications, from thermonuclear fusion in tokamak reactors to astrophysical models. These models are characterized by a nonlinear system of partial differential equations in which the flow of the fluid strongly couples to the evolution of electromagnetic fields. In this talk, we consider the one-fluid, viscoresistive MHD model in two dimensions. There have been numerous finite-element formulations applied to this problem (e.g. [1, 2, 3]), and we will briefly discuss the applications of two; a least-squares and mixed-method formulation. In the latter, we consider inf-sup stable elements for the incompressible Navier-Stokes portion of the formulation, Nedéléc elements for the magnetic field, and a second Lagrange multiplier added to Faraday's law to enforce the divergence-free constraint on the magnetic field.

Regardless of the formulation, the discrete linearized systems that arise in the numerical solution of these equations are generally difficult to solve, and require effective preconditioners to be developed (e.g. [5]). Thus, the final portion of the talk, will involve a discussion of monolithic multigrid preconditioners, using an extension of a well-known relaxation scheme from the fluid dynamics literature, Vanka relaxation, to this formulation [4]. To isolate the relaxation scheme from the rest of the multigrid method, we utilize structured grids, geometric interpolation operators, and Galerkin coarse grid operators. Numerical results are shown for the Hartmann flow problem, a standard test problem in MHD.

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