# An Accurate Solution of a Singular Thermoplastic Problem of Pressure-Dependent Plasticity

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## ABSTRACT

It is known that thermorigidplastic problems based on the double shearing model [1] are singular if the maximum friction law is adopted as one of the boundary conditions [2]. In particular, some components of the strain rate tensor and, therefore, the plastic work rate approach infinity in the vicinity of maximum friction surfaces. On the other hand, the plastic work rate is involved in the heat conduction equation. Therefore, conventional finite element methods are not capable of solving such boundary value problems. In [2], stationary flow is considered. In the case of non-stationary flow the behaviour of solutions in the vicinity of maximum friction surfaces is more completed and no general analysis is available in the literature.

This paper presents a theoretical investigation into heat generation in the continued quasi-static plane strain compression of a thin metal strip between two rigid, parallel perfectly rough dies. The strip material obeys the doublé shearing model. The length of the dies is supposed to be much greater than the current strip thickness. The máximum friction law is assumed at the die surface. In the case of the doublé-shearing model this means that an envelope of characteristics coincides with the friction surface. The solution is given in Lagrangian coordinates. Using these coordinates significantly facilitates the solution. In particular, the original initial/boundary value problem reduces to the standard second initial/boundary value problem for the nonhomogeneous heat conduction equation. Therefore, the Green's function is available in the literature. This function is used to find the solution. However, the heat conduction equation contains a singular term. Therefore, an auxiliary function of temperature is introduced. Then, the heat conduction equation becomes non-singular and its solution can be found with a high accuracy.

The distribution of temperature and heat flux in the vicinity of the maximum friction surface is investigated for various boundary conditions on temperature and heat flux. The effect of pressure – dependency of the yield criterion on this distribution is revealed.

## REFERENCES

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