A Stable and Efficient Domain Decomposition Method for Maxwell's Equations with Uncertainty

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Large-scale permanent storage of CO_2 under the seafloor has been identified as an important means to mitigate the effects of global warming by reduction of atmospheric emissions [1]. To safely utilize the subsurface CO_2 storage sites, there is a great need to control risk factors, e.g., leakage risks and over-pressurization. The amount of known data in a typical storage site is small compared to the vast spatial extent (1000-100,000 km^2). This leads to significant uncertainties. In order to understand and control the risk factors, a combination of flow simulations and geophysical monitoring techniques must be applied. In controlled-source electromagnetics, signals are transmitted into the subsurface reservoir. The returning signals are measured at the surface level and sequentially incorporated into the geological model to update the CO_2 migration path and the pressure build-up. Numerical simulation of this problem requires efficient numerical methods for Maxwell's equations that can handle uncertainties and large domains. We will present a spatially adaptive finite difference method for a stochastic formulation of Maxwell's equations.

To include uncertainty in Maxwell's equations, we will use the stochastic Galerkin method and the polynomial chaos framework [2]. The Maxwell's equations are projected onto stochastic basis functions. The resulting extended systems need to be solved only once to obtain all statistics of interest, such as best estimates of monitoring scenarios, or confidence intervals on leakage risks.

We use high-order finite-difference methods that satisfy a summation-by-parts rule for the spatial discretization of Maxwell's equations [3, chapter 7]. Typically, a very fine resolution is required close to sources and receivers compared to in the farfield. In order to satisfy the different resolution requirements in different parts of the spatial domain, we incorporate spatial adaptivity by decomposing the spatial domain into structured mesh blocks. The discretization of each block is adapted to the local resolution requirement. We prove the spatial discretization to be time-stable by a carefully designed numerical coupling between mesh blocks of different grid sizes. The combination of spatial adaptivity and high-order finite difference methods on block-structured grids leads to a highly efficient and accurate numerical discretization.

References

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