On Couplings of the Compressible and Incompressible Navier-Stokes Equations

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Abstract

Gas-liquid multiphase flows occur in numerous situations such as injection and combustion of fuel droplets, shock induced mixing of liquids, cloud cavitation and explosive volcanic eruptions. Understanding the dynamics of these problems are rather demanding and can be handled with three different methodologies: one can treat both phases as compressible, both phases as incompressible or the gas as compressible and the liquid as incompressible.

The fully compressible model is limited to liquids for which there are acceptable models for their compressible evolution [1]. The fully incompressible model is valid as long as the density variations in the gas phase can be neglected [2]. These two models lead to straightforward ways of coupling, because they couple the different parts of the system with the same mathematical model. The compressible-incompressible coupling is more realistic and general, but more complicated than the previous ones, since the mathematical models are different [3, 4].

In this paper, we consider the last approach and aim for interface conditions such that the coupled problem is well-posed. Next, we discretize using high order finite differences on summation-by-parts form and include the derived coupling conditions weakly. To the best of our knowledge, such a scheme has not been proposed before. We will follow the general procedure derived in [5], where the energy-method is the main analytical tool.

References


