Hierarchical Isogeometric Model Reduction Applied to Data Assimilation

Yves Antonio B.C. Barbosa*, Simona Perotto* and Alessandro Veneziani♭

* MOX, Department of Mathematics, Politecnico di Milano
Piazza Leonardo da Vinci 32, I-20133 Milano, Italy
e-mail: yvesantonio.brandes@polimi.it, simona.perotto@polimi.it

♭ Department of Mathematics and Computer Science
400 Dowman Dr, 30322 Atlanta, GA, USA
e-mail: ale@mathcs.emory.edu

ABSTRACT

The progressively extended use of scientific computing in many fields of engineering and life sciences demands an accurate assessment of numerical modeling reliability. Almost invariably, the models of interest are represented by partial differential equations featuring parameters reflecting the constitutive laws used and critical to the numerical results. In many cases, those parameters are unknown in the applications and require an appropriate estimation. The latter can be rigorously pursued by "Data Assimilation" (DA) techniques, where the parameters of interest are used to minimize an appropriate quantification of the mismatch between available measures of outputs and the results of numerical simulations. One of the drawbacks of this procedure is generally the computational cost intrinsic to the constrained minimization nature of the procedure, the constraints being in general a system of partial differential equations. A possible approach consists of using reduced models for solving the equations required by the iterative minimization procedures.

In this presentation, we investigate the impact of applying a Hierarchical Model reduction technique [1] combined with Isogeometric Analysis (HigaMod) to data assimilation. HigaMod is a reduction procedure ideal to downscale phenomena characterized by a preferential direction, e.g., when modelling the blood flow in arteries. The idea is to simplify the model by decoupling the dominant and transverse dynamics, using different discretization techniques. In particular, an isogeometric approach is used to describe the main stream, while a modal expansion is employed to model transverse dynamics [2]. Since the modal expansion can be tuned according to the local complexity of the transverse dynamics [3], the number of degrees of freedom required to achieve a certain accuracy on the solution can be significantly reduced. This represents an ideal scenario for multi-query simulations, i.e., to apply DA algorithms for multiple parameter estimation. We focus on unsteady advection-diffusion-reaction models. Minimization is pursued in a stochastic setting, within a Kalman filtering approach. Preliminary results point out that the computational saving led by HigaMod approach is apparent and however it does not compromise the reliability of the DA procedure. This work is supported by NSF DMS 1412963.

REFERENCES

[1] S. Perotto. *A survey of Hierarchical Model (Hi-Mod) reduction methods for elliptic problems.* In Numerical Simulations of Coupled Problems in Engineering. Series: Computational Methods in Applied Sciences, Vol. **33**, Springer, S.R. Idelsohn Ed., 217–241, 2014.

[2] S. Perotto, A. Reali, P. Rusconi, and A. Veneziani. *HIGAMod: a Hierarchical IsoGeometric Approach for MODel reduction in curved pipes.* Accepted for the publication in Comput. & Fluids, 2016.

[3] S. Perotto, and A. Veneziani. *Coupled model and grid adaptivity in hierarchical reduction of elliptic problems.* J. Sci. Comput., **60**(3): 505–536, 2014.