Asymptotic limits of the lattice Boltzmann phase field model based on the conservative Allen-Cahn equation

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ABSTRACT

Advective phase field models can be used for the modelling of multi-phase flow when coupled to an appropriate Navier Stokes solver like the lattice Boltzmann method. Geier, Fakhari and Lee [1] recently proposed a phase field model based on the conservative Allen-Cahn equation that lends itself to efficient discretization with the lattice Boltzmann model such that both the phase field model and the fluid solver use the same algorithmic structure. One particular attractive feature of the model is that it can be implemented in way that preserves the locality of the lattice Boltzmann collision step by computing the phase gradient through the first moments of the phase field distribution function. It is however observed that this degrades the accuracy of the model compared with computation by finite differences.

Recently Wang et al. [2] and Ren et al. [3] independently proposed identical modifications to the model in order to enhance its accuracy. Both proposals are based on the Chapman-Enskog multi-scale expansion while the derivation of the original model by Geier et al. was based on an asymptotic expansion under diffusive scaling.

In this contribution we revisit the analysis techniques and discuss why they lead to different results. It is noted that the asymptotic analysis of phase field models is difficult due to the representation of a singularity (the phase interface) by a smooth function. One important question is how the Cahn Number (ratio of interface width to characteristic length of the flow) is scaled together with the grid spacing and the time step. In practical simulations the grid-Cahn number (ratio of interface width to grid spacing) is kept constant while in the asymptotic analysis the Cahn number was kept constant. This discrepancy is based on the fact that when keeping the grid-Cahn number constant, as done by Wang et al., the Taylor series expansion of the phase field equation does not converge and no asymptotic limit exists.

We find that the original method is asymptotically correct in the appropriate limit and that the correction proposed by Ren et al. and Wang et al. are of the same order as the truncation error (which does not mean that they are not useful). On the other hand, we observe that the Chapman-Enskog expansion with constant Mach number and constant grid-Cahn number requires additional assumptions for the truncation of the involved Tayler series, which are not easily justified on theoretical grounds.

REFERENCES

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