Mixed variational formulations for multi-field problems

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ABSTRACT

In the present contribution we focus on novel computational formulations for multi-field coupling in large strain problems. In particular, polyconvex large strain thermoelasticity as well as polyconvex large strain phase-field fracture formulations are considered.

The formulations of both multi-field problems are based on an extended kinematic set, the deformation gradient (line map), its co-factor (area map) and its determinant (volume map), and their conjugate stresses. The strain measures are defined via the tensor cross product between second order tensors as proposed in Bonet et al. [1]. Thereby, the linearization of the systems can be expressed in compact form and the conjugate stresses relate to classical stress tensors in elegant manner. Moreover, the formulations allow for the introduction of mixed variational principles in a straightforward manner.

For the thermomechanical system, a Hellinger-Reissner type variational principle based on the definition of a complementary energy density function is applied. Therein, the deformed geometry and the absolute temperature as primal variables are supplemented by the newly introduced conjugate stresses. The thermomechanical coupling is directly implemented between the temperature and the conjugate stress to the Jacobian determinant, see Dittmann [2].

Variationally consistent formulated phase-field methods to fracture are able to predict complex three-dimensional crack patterns. Here, we propose a novel approach based on a mixed Hu-Washizu formulation, where the primal variables are given by the deformed geometry, the extended set of strains and conjugate stresses as well as the phase-field variable, see Hesch et al. [3].

This promising approaches enable great flexibility and clarity for the development of new multi-field formulations, since the constitutive laws can be solved as separate field. This can be done without enlarging the system to be solved using suitable static condensation procedures, hence, no additional computational effort for the solution is required. To this end, we apply discontinues constant and linear interpolations to the stresses and/or strains, whereas the displacement field and the additional field of the corresponding multi-field problem is discretized ultilizing quadratic B-spline based shape functions. The chosen approximations ensure optimal convergence.

REFERENCES

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