Enhancement of POD-Galerkin Reduced Order Models (ROM) for the 3D-unsteady and turbulent incompressible Navier-Stokes equations

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ABSTRACT

The size and complexity of 3D unsteady simulations of aeronautical engines have been steadily increasing and the exploitation of such simulations has become a challenge. The Snapshots POD applied to the flow velocity field guarantees the existence of a low dimensional space, which captures the large energetic scales of turbulence. Nevertheless, it is known that the small scales responsible for the dissipation of Turbulent Kinetic Energy (TKE) appear only within the neglected POD modes. In [1], an a priori implementation of a traditional eddy-viscosity based closure model is proposed. In this work, we propose a new stabilization approach for the POD-Galerkin projection of the unsteady incompressible and turbulent Navier-Stokes equations thanks to an extended and minimal POD subspace in order to recover correctly the assessment of the High Fidelity TKE. The key for this, is the enrichment of the flow velocity POD subspace, by POD modes associated with the spatial gradient of the flow velocity. A solution for the scaling issue due to the combination of POD velocity and POD gradient modes is proposed.

We denote $X = [L^2(\Omega)]^3$ and Ω a bounded 3D open set. The inner product is the kinetic energy-based one associated with the X-norm, denoted respectively by $(:,:)$ and $\|.\|$. Let $v(t)$ the velocity field of an unsteady incompressible flow. We consider the POD based on the velocity gradient by the snaphsots technique: $\Psi_n = \frac{1}{\sqrt{M}} \sum_{i=1}^{M} B_{i,n} \nabla v(t_i)$. Where $B_n = (B_{i,n})_{1 \leq i \leq M}$ for $n = 1, ..., M$, a set of orthonormal eigenvectors of the temporal correlations matrix: $(\nabla v(t_i), \nabla v(t_j))_{[L^2(\Omega)]^9}$, $i, j = 1, ..., M$. The dissipative POD modes for the velocity are defined accordingly based on the same set of orthonormal eigenvectors B_n: $\Phi_n^E = \frac{1}{\sqrt{M}} \sum_{i=1}^{M} B_{i,n} v(t_i)$. For a 3D turbulent flow configuration presented in Fig. 1(a), Ψ_1 is shown in Fig. 1(b). Interestingly, the velocity field of the most dissipative POD mode features high scales in the shear layers and in the wake of the two triangular obstacles. The dissipative POD modes embedded onto a reduced N dimensional space generated by a POD Φ based on the velocity field v, do not carry any TKE as illustrated in Fig. 1(c). So, the total dissipation of TKE is largely under-estimated in POD Galerkin ROM if no dissipative modes are introduced. In order to perform the reduction, the dissipative modes Φ_n^E have to be incorporated to the set of POD modes Φ_n leading to a single set of orthonormal modes, onto which a classical Galerkin projection of the Navier-Stokes equations is performed, and an evaluation of its accuracy will be proved before and after applying the proposed enrichment.

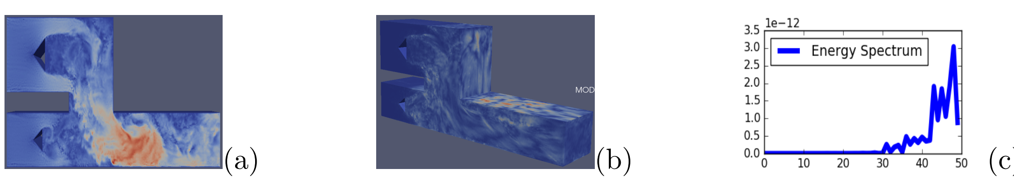

Figure 1: (a) Instantaneous velocity magnitude of a 3D unsteady simulation of a turbulent flow at $Re = 80'000$. (b) Ψ_1. (c) $\left\| \sum_{n=1}^{N} (\Phi_m^E, \Phi_n) \Phi_n \right\|^2 / \left\| \Phi_m^E \right\|^2$ $N << M$, for $m = 1, ..., 50$.

REFERENCES

[1] M. Balajewicz et al./Journal of Computational Physics, Vol. 321, 2016.