# A multi-scale model for mass transport in arteries and tissue

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## ABSTRACT

Mathematical models have become more and more important in many applications from medicine and biology. Quite often the resulting system of equations is complex, highly non-linear and cannot be solved analytically. Thus stable and robust numerical simulation methods play an important role in understanding phenomena like the regulatory mechanisms of the heart or the balancing of metabolic supply and demand. By these non invasive techniques, physiological processes in the human body can be examined and understood with less effort and less danger for a patient. As a consequence, scientists can get more insight into the inherent mechanisms and improve their diagnosis techniques. Moreover quantitative prediction of the distribution of a chemical compound in living tissues, which provide important contributions to the development of new medical products [1], will be possible by the help of numerical simulation. In addition to that the impact on organs suffering from a reduced supply of oxygen can be predicted by the help of computational methods. A well known problem in this field is, e.g., to estimate the risk of ischemia caused by a stenosis [2]. Our aim is to give a reliable prediction of the distribution of certain chemicals like oxygen, carbon dioxide or lactate during the transport in blood vessels and human tissue.

Since blood flow within the network is fast compared to the flow within the tissue, we apply a domain decomposition in such a way that we separate the vessel network and the porous tissue and assign different models to them [1]. Blood flow and transport processes within the porous medium are governed by Darcy's equation and a convection-diffusion equation in 3D. To model the network flow and transport, 1D reduced models and 0D lumped parameter models are used, which are given by transport equation systems or ODE-systems, respectively [1-4]. All in all we have a coupled 0D-1D-3D problem to describe the hierarchical flow and transport from the heart to the capillaries and tissue. In this talk we will discuss the model and appropriate discretization techniques in detail. Finally, some simulation results illustrating the impact of a stenosis on tissue perfusion and oxygen supply are presented.

## REFERENCES

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