DtN-Based Mixed-Dimensional Coupling Using a Boundary Stress Recovery Technique

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ABSTRACT

The need to reduce the size of large discrete models is a reoccurring theme in computational mechanics in recent years. One situation which calls for such a reduction is that where the solution in some region in a high-dimensional (highD) computational domain behaves in a low-dimensional (lowD) way. Typically, this situation occurs when the lowD model is employed as an approximation to the highD model in a partial region of the spatial domain. The highD region is the region of interest where the lowD approximation does not hold or does not provide the details required from the analysis, whereas the lowD solution is possibly of less interest in its details to the analyzer, yet it affects the solution in the highD region significantly. In our present study, the coupling of two-dimensional (2D) and one-dimensional (1D) models in time-harmonic elasticity is considered. The hybrid 2D-1D model is justified in case that some regions in the 2D computational domain behave approximately in a 1D way. This hybrid model, if designed properly, is much more efficient than the standard 2D model taken for the entire problem. Other fields of application where the coupling scenario is of special interest include, among others, blood-flow analysis, hydrological and geophysical flow models. Two important issues related to such hybrid 2D-1D models are (a) the design of the hybrid model and its validation (with respect to the original problem), and (b) the way the 2D-1D coupling is done, and the coupling error generated. Our research focuses on the second issue. Several numerical methods are adapted to the 2D-1D coupling scenario, for elastic time-harmonic waves. All are existing methods that deal with interfaces; however none of them has previously been adopted and applied to the type of problem under study here. Among these methods there are coupling techniques where a stress calculation stands on their basis, and affects their accuracy significantly. Such techniques are *Dirichlet-to-Neumann (DtN) map*, *Landis* method and an *iterative DtN* method. When using numerical schemes, such as finite elements, the calculation of the solution function or derivative may use the benefit of a better approximation. Post-processing procedures, with recovery techniques among them, were developed in order to fill that need. We have modified a boundary-stress-recovery (*BSR*) process, originally proposed by Hughes, which recovers the order of accuracy of the FE primary variable. The technique was implemented to the mentioned coupling methods. The usage of the *BSR* lead to an improvement of up to two orders in the relative error, computed by comparing the solution generated by a hybrid model to a reference solution generated by a truly 2D model. The accuracy of the 2D-1D coupling by the methods is compared numerically for a specially designed benchmark problem, and conclusions are drawn on their relative performances. Other problems with higher level of complexity were solved as well using these coupling methods. All examined methods demonstrate very good performances in all investigated cases. The accuracies of the *DtN* method and *Landis* method are similar, while the precision achieved using the iterative method depends on the chosen convergence criteria, where a tight value of the latter can lead to excellent results, accompanied by a relatively large computational effort.

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