A Reduced Basis Method for heterogeneous domain decomposition problems

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ABSTRACT

The Reduced Basis method allows efficient model reduction of parametrized partial differential equations. It is a snapshot-based method that builds upon a high-dimensional “detailed” approximation method, such as the Finite Element or the Finite Volume method. By the widely used Greedy-algorithm a set of sample solutions is generated, that spans the Reduced Basis approximation space. After that, a Galerkin projection yields the reduced solution. The efficiency of the method highly relies on a decomposition of the computations into an offline- and online-phase and on a-posteriori error estimates, that allow to estimate the error induced by the reduction.

A critical issue in Reduced Basis methods for domain decomposition problems is to find functions that approximate the traces of the detailed solutions on the interface. There have been several efforts including an eigenvalue decomposition [1], “empirical” basis functions [2] and “fourier” interface functions [3]. In our work we consider a framework, that treats the approximation on the interface as a black box. In this way, we can treat heterogeneous domain decomposition problems as opposed to previous works. We also present a-posteriori error analysis for problems that include coercive as well as non-coercive subproblems.

Numerical results are demonstrated with a coupled Stokes-Darcy model as a scenario for a groundwater application. We demonstrate the flexibility of the method with respect to the number of interface basis functions and show that our approach yields an efficient and accurate approximation if few global “detailed” solutions are computed in the offline-phase. We also observe, that the a-posteriori error estimation can be problematic when dealing with heterogeneous problems.

REFERENCES