New Purely Hyperbolic Advection-Diffusion Models for the Numerical Formulation of Coupled Problems

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ABSTRACT

From the theoretical point of view it seems quite obvious that diffusive processes cannot take place at infinite velocity inside matter, since this fact would violate causality in the special relativity framework. Moreover, there is some experimental evidence in favour of this assertion [1].

However, the standard diffusion equation is based on Fick's law in the case of mass transport, and on Fourier's law in the case of heat conduction. Some other quite common constitutive laws of mathematical physics establish a similar time-independent functional relation between the flux and the spatial gradient of the state variable. This is the case of Ohm's law in electricity, of Darcy's law in fluid motion within porous media and of Stokes' stress law in CFD.

When combined with the adequate conservation principle, these type of constitutive laws lead to parabolic-type mathematical models that predict an infinite speed of propagation for the mass, energy, charge, momentum or whatever species is being transported. For this reason, the infinite speed paradox underlies the vast majority of mathematical models that are used in computational mechanics for solving engineering and science problems.

It happens to be ironical that the first comprehensive derivation for the standard diffusion equation was given by Einstein himself [2]. In fact, Einstein recognized this result to be physically unrealistic and pointed out the need to modify the diffusion equation in order to overcome the paradox.

In this paper we present a new Cattaneo-type constitutive law for diffusion that generalizes the one that had been formerly proposed by the authors [3,4]. The equations of this kind overcome the infinite speed paradox and take care for new phenomena (diffusion waves), while they yield stable results in advection dominated situations.

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