A structure-preserving integrator for thermo-elastic rods

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ABSTRACT

A new structure-preserving and thermodynamically consistent time integrator for simulating the two-dimensional nonlinear dynamics of geometrically exact rods able to conduct heat is developed. We carry out the simultaneous discretization in space and time of Hamilton's variational principle to formulate a new explicit variational integrator in the adiabatic case. The method is explicit, second-order accurate and can be identified with a Lie-group symplectic partitioned Runge-Kutta method for finite element discretizations of thermoelastic rods involving large rotations and displacements. Additionally, heat conductions is considered by means of adding a dissipative but otherwise equivariant flux to the entropy balance equation. The discrete version of D'Alembert principle is then used to formulate the new integration methods. The resulting algorithm is (i) momentum-conserving in the sense that it exactly conserves the momenta asoociated to symmetries of the system, (ii) thermodynamically consistent since for isolated systems, the total energy is nearly exactly conserved for exponentially long periods of time and the total entropy is non-decreasing and, (iii) symplectic in the non-conductive limit. Numerical examples allow to verify that the properties of the algorithm.

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