Angular Momentum preserving cell-centered Lagrangian and Eulerian schemes on arbitrary grids

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ABSTRACT

This talk is dedicated to multi-dimensional numerical simulation of Euler equations for compressible flow problems. To fulfill the conditions of the Lax-Wendroff theorem, it is required that a numerical scheme conserves the mass, momentum and total energy. Associated with the consistency and the increase of the entropy, these conservation properties are sufficient, in 1D, to ensure that a converged solution of the scheme is an entropic solution of the Euler equations. However, such theoretical result has not been extended to multi-dimension. Numerous studies (refer for instance to [1-3]) tend to show that the conservation of a forth quantity, the angular momentum, plays an important role in multi-dimensional simulations.

We address the conservation of angular momentum for cell-centered discretization [4,5] of compressible fluid dynamics on general grids. We focus on the Lagrangian step which is also sufficient for Eulerian discretization using Lagrange+Remap. Starting from the conservative equation of the angular momentum, we show that a standard Riemann solver (a nodal one in our case) can easily be extended to update the new variable. This new variable allows reconstructing all solid displacements in a cell, and is analogous to a partial Discontinuous Galerkin (DG) discretization. We detail the coupling with a second-order Muscl extension. All numerical tests show the important enhancement of accuracy for rotation problems, and the reduction of mesh imprinting for convergent flows. The generalization to the axi-symmetric case is detailed.

[1] S. Driscoll, “The Earth’s Atmospheric Angular Momentum Budget and its Representation in Reanalysis Observation Datasets and Climate Models”, *PhD University of Reading, School of Mathematical and Physical Sciences* (2010).

[2] A. Oort, “Angular Momentum in the Atmosphere-Ocean-Solid Earth System”, *Bulletin of the American Meteorological Society*, 70 (10), 1, 231-1242 (1989).

[3] P. Roe and B. Morton, “Preserving vorticity in finite-volume schemes”, *Finite volumes for complex applications II*, 347-356, Hermes Sci. Publ., Paris (1999).

[4] B. Després and C. Mazeran, “Lagrangian gas dynamics in 2D and lagrangian systems”, *Arch. Rat. Mech. Anal.*, 178:327-372 (2005).

[5] P.H. Maire, R. Abgrall, J. Breil and J. Ovadia, “A cell-centered Lagrangian scheme for 2D compressible flow problems”, *Siam. J. Sci. Comput.*, 29 (2007).