Parametric Identification of Mathematical Models of Coupled Thermo-Elasticity Problem

Aleksey V. Nenarokomov*, Pavel V.Prosunysov† and Dmitry M. Titov*

*Dept. of Space System Engineering
Moscow Aviation Institute (Technical University)
Volokolamskoe Hgw. 4, Moscow, 125993, Russia
e-mail: Aleksey.nenarokomov@mai.ru, web page: http://www.mai.ru

† Dept. Rocket and Space Composite StructureSM-10
Moscow State Technical University
2nd BaumanStr., 5,105005, Moscow, Russia
Email: prosutsov@tochka.ru

ABSTRACT

In many practical situations it is impossible to measure directly such characteristics of analyzed materials as thermal and elastic properties. The only way, which can often be used to overcome these difficulties, is the solution of inverse problems of mathematical physics. Such problems are ill-posed and special regularizing methods are needed to solve them. The final aim of this paper is to estimate thermal and elastic properties of advanced materials using the approach based on inverse methods (as example: thermal conductivity $\lambda(T)$, heat capacity $C(T)$ and heat expansion coefficient $\alpha(T)$). The system of covering equation is

$$C(T)\frac{\partial T(x,y)}{\partial \tau}=\frac{\partial}{\partial x}(\lambda_x(T)\frac{\partial T(x,y)}{\partial x})+\frac{\partial}{\partial y}(\lambda_y(T)\frac{\partial T(x,y)}{\partial y});$$

$$\tau=0;\quad T(x,y)=T_0(x,y);\quad \Gamma\in\Gamma_1;\quad -\lambda_y\frac{\partial T(x,y)}{\partial y}=-\varepsilon_w\sigma(T(x,y)^4-T_f^4);$$

$$\Gamma\in\Gamma_2;\quad -\lambda_x\frac{\partial T(x,y)}{\partial x}=-\varepsilon_w\sigma(T(x,y)^4-T_f^4);\quad \Gamma\in\Gamma_3;\quad -\lambda_y\frac{\partial T(x,y)}{\partial y}=0;$$

$$\Gamma\in\Gamma_4;\quad -\lambda_n\frac{\partial T(x,y)}{\partial n}=q_{res}(\tau);\quad \Gamma\in\Gamma_5;\quad -\lambda_x\frac{\partial T(x,y)}{\partial x}=0,$$

$$E_{11}\frac{\partial^2 u(x,y)}{\partial x^2}+(E_{12}+G_{xy})\frac{\partial w^2(x,y)}{\partial x\partial y}+G_{xy}\frac{\partial^2 u(x,y)}{\partial y^2}+\frac{\partial}{\partial x}(a_1(T)\theta(x,y))=0;$$

$$(E_{21}+G_{xy})\frac{\partial^2 u(x,y)}{\partial x\partial y}+E_{22}\frac{\partial w^2(x,y)}{\partial y^2}+G_{xy}\frac{\partial^2 w(x,y)}{\partial x^2}+\frac{\partial}{\partial y}(a_2(T)\theta(x,y))=0;$$

$$E_{11}=\frac{E_x(1-\nu_{yz}^2)}{(1-\nu_{xz}^2)(1-\nu_{yz}^2)-(\nu_{xy}+\nu_{xz}\nu_{yz})^2};\quad E_{12}=\frac{E_x(\nu_{xy}+\nu_{xz}\nu_{yz})}{(1-\nu_{xz}^2)(1-\nu_{yz}^2)-(\nu_{xy}+\nu_{xz}\nu_{yz})^2};$$

$$a_1(T)=\frac{E_x\left((\alpha_x(T)+\alpha_z(T)\nu_{xz})(1-\nu_{yz}^2)+(\alpha_y(T)+\alpha_z(T)\nu_{yz})(\nu_{xy}+\nu_{xz}\nu_{yz})\right)}{(1-\nu_{xz}^2)(1-\nu_{yz}^2)-(\nu_{xy}+\nu_{xz}\nu_{yz})^2};$$

$$E_{21}=\frac{E_y(\nu_{xy}+\nu_{xz}\nu_{yz})}{(1-\nu_{xz}^2)(1-\nu_{yz}^2)-(\nu_{xy}+\nu_{xz}\nu_{yz})^2};\quad E_{22}=\frac{E_y(1-\nu_{xz}^2)}{(1-\nu_{xz}^2)(1-\nu_{yz}^2)-(\nu_{xy}+\nu_{xz}\nu_{yz})^2};$$

$$a_2(T)=\frac{E_y\left((\alpha_y(T)+\alpha_z(T)\nu_{yz})(1-\nu_{xz}^2)+(\alpha_x(T)+\alpha_z(T)\nu_{xz})(\nu_{xy}+\nu_{xz}\nu_{yz})\right)}{(1-\nu_{xz}^2)(1-\nu_{yz}^2)-(\nu_{xy}+\nu_{xz}\nu_{yz})^2},$$

Such problems are of great practical importance in the study of properties of materials used as non-destructive surface coating in objects of space engineering, power engineering etc.