Versatile approach for preconditioning linear systems for the solution of N-field coupled problems

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ABSTRACT

We propose a flexible framework for building algebraic multigrid (AMG) preconditioners for linear systems arising from coupled problems. The preconditioners are constructed using external input files without the need of recompiling the source of the underlying multiphysics code. This approach allows the final users having access to a high variety of block preconditioners through a flexible high level interface, and rapidly prototyping precondition techniques for challenging multiphysics problems. The set of block preconditioners that can be built with such an approach accounts for most of the alternatives available in the literature.

Two main preconditioner types are considered. The first type consists of techniques similar to the ones proposed in [1]. A block method (such as block Gauss-Seidel or an incomplete block factorization) is considered as an outer iteration for handling the coupling between the fields, and then, single field solvers (in particular AMG) are used to solve the resulting single field problems. The proposed approach is general for N-by-N block systems because incomplete block factorizations can also be handled with N greater than 2. This is achieved with a block recursive strategy similar to the one proposed in [2]. This first type of preconditioners treats the coupling only at the finest level: the multigrid hierarchies used inside the single fields are uncoupled.

The second type includes preconditioners similar to the ones introduced in [3], which are able to handle the coupling at all multigrid levels. This approach requires building coarse level representations of the block matrix, which are obtained using only the multigrid hierarchies of the single fields. The block matrices are processed using block methods (such as block Gauss-Seidel or an incomplete block factorization) at each multigrid level.

Both preconditioner types require only multigrid hierarchies for the individual fields which can be generated using available AMG packages such as MueLu [4]. The proposed framework is implemented in the multiphysics code BACI [5].

The approach is successfully applied to time-dependent thermo-structure interaction and time-dependent fluid-structure interaction. Further problem types are going to be considered.

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