Numerical modeling of non-Newtonian biomagnetic fluid flow

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ABSTRACT

The idea that external magnetic fields interact with biofluids and alter their flow behaviour is relatively new and lead to the emergence of biomagnetic fluid dynamics (BFD, Haik *et al.* [1]) with abundant applications in bioengineering and the medical sciences. The most common biofluid is blood, with its magnetic behavior stemming from the presence of iron oxide in the hemoglobin molecule in red blood cells. The Newtonian rheological assumption for blood although widely adopted in the literature may become problematic under low shear rate flow conditions which have been associated with the development of vascular disease.

To this end, we extend and validate the pressure correction scheme developed by Botti and Di Pietro [2] to account for non-Newtonian incompressible flux flow simulations. The scheme, originally developed for Newtonian fluids, consists of a combined discontinuous Galerkin approximation for the velocity and a continuous Galerkin approximation for pressure. In order to account for the varying viscosity, the symmetric weighted interior penalty (SWIP) formulation is employed. The stress tensor is computed explicitly, disregarding its non-linearity in the Jacobian computation. Even though this strategy yields an approximated Jacobian, the convergence rate of the Newton iteration is not significantly affected and the computational efficiency of the scheme is retained.

The proposed scheme is utilized in order to demonstrate the effects of magnetic fields on biomagnetic fluid flows. For the most general case a full coupling of the Navier-Stokes and Maxwell equations must be carried out [1]. However, if the time variation of the electric field in Ampere's law is ignored and Ohm's law is used to eliminate the electric field, an induction equation for the magnetic field can be derived. The induction equation is coupled with the Navier-Stokes equations in the MHD approximation. In this work, the flow of an electrically conducting magnetic biofluid under the influence of a time independent magnetic field is described by setting,

$$\mathbf{f} = \mathbf{J} \times \mathbf{B} + \mu_0 \left(\mathbf{M} \cdot \nabla \right) \mathbf{H},$$

as a body force in the Navier-Stokes equations. The effect of externally applied magnetic fields on the flow of Newtonian and non-Newtonian fluids through a straight rigid tube with axisymmetric stenosis is compared. Both steady and time varying flows are considered. The proposed scheme can have important applications in blood flow control.

REFERENCES

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