Effective stress in unsaturated granular materials: micro-mechanical insights

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ABSTRACT

The mechanical behaviour of granular materials with low moisture content is micro-mechanically analyzed. Such materials involve a mixture of three coupled phases (gas, liquid, and solid grains), which leads to a drastically different behaviour from two-phase systems (solid, and liquid or gas phases). Low degrees of saturation are considered, so that the three-phase medium can be considered in the pendular regime, with the liquid phase forming distinct menisci that join solid grain pairs. These menisci introduce interfaces, inducing attractive *capillary forces* that act on the grains to produce complex constitutive behaviours.

The various internal forces that arise in such a three-phase medium are analytically derived according to liquid bridge distribution and their geometrical properties, as well as grain packing. As such, a capillary stress tensor is obtained that distinguishes the total stress tensor from the grain-grain internal forces, i.e. the so-called *effective stress tensor*. The resulting analytical expressions highlight the non-spherical nature of the capillary stress tensor, whereas most studies in the literature are empirically based and assume moisture action to be isotropic, e.g. [1]. Furthermore, distinct contributions to this newly derived capillary stress tensor arise in our derivations. The first one relates to the unequal loading applied onto the solid grains by the gas and liquid phases with a pressure difference known as matric suction. The second contribution is due to contractile skin effects where surface tension forces act along wetted contours of grains because of liquid-gas interfaces [2]. These two contributions are analyzed in detail, focusing on their anisotropic features as a function of grain packing and liquid bridge distribution.

Discrete numerical simulations show the relevancy of such derived expressions. A discrete element method based computational model involving capillary forces [3] is used to simulate various loading paths. Classical axisymmetric paths ($\sigma_2 = \sigma_3 = \text{cst}$), as well as proportional stress ($d\sigma_1 = R d\sigma_2 = R d\sigma_3$) and simple shear paths, are applied to mono- and poly-dispersed granular samples with periodic boundary conditions. It is shown that the derived effective stress is the adequate variable to express the strength of the unsaturated material. Indeed, using this effective stress, a unique plastic limit criterion appears, for all loading paths and saturation degrees, including the dry case. The relevancy of this effective stress tensor for constitutive modelling, i.e how it relates to deformations, is also discussed.

REFERENCES

[1] A. W. Bishop, "The principle of effective stress", *Teknisk Ukeblad*, vol. 106, pp. 859–863, 1959.

[2] R. Wan, S. Khosravani and M. Pouragha, "Micromechanical Analysis of Force Transport in Wet Granular Soils", *Vadose Zone Journal*, vol. 13(5), pp. 1–12, 2014.

[3] L. Scholtès, B. Chareyre, F. Nicot and F. Darve, "Micromechanics of granular materials with capillary effects", *International Journal of Engineering Science*, vol. 47(1), pp. 64–75, 2009