Temperature-based thermodynamically consistent time integration for nonlinear thermoelasticity

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ABSTRACT

This work is concerned with structure preserving integrators for evolution equations involving thermoelasticity or thermoviscoelasticity in dynamical problems. Preserving structure integrators are widely developed for conservative (Hamiltonian) systems, being the most representative method the well-known *energy-momentum* due to Simó and Tarnow [1]. Recently, after the works of Romero I. [2] and [4], these ideas are being used in evolution system with irreversible processes, i.e non-conservative, such as those with internal dissipation mechanisms (viscoelasticity, plasticity, damage). In this context, the structure meant to be preserved is understood in the thermodynamical way, namely, the integrator must intrinsically satisfy the laws of thermodynamics (plus symmetries of the system if they exist). The theoretical key ingredient behind the new procedure to derive TC integrators relies on the GENERIC form of the evolution equations of the thermodynamical system at hand.

Within this framework, we present a novel integration method for the dynamic of a nonlinear threedimensional thermoelastic/thermoviscoelastic continuum body using the temperature as thermodynamical state variable. So far, the GENERIC-based TC integrators newly proposed have been based on the entropy as thermodynamical state variable, see [2,3,4], since it was reported to best suit the departing point of GENERIC formalism, as indicated in [2]. However, this choice obliges to assume important constrains to the formulation, such as the necessity for material models to enable the analytical provision of its potentials in terms of the entropy and, more importantly, the incapability to impose Dirichlet's boundary conditions. That is why, the use of temperature becomes crucial to succeed in dealing with Dirichlet's boundary conditions usually appearing in many engineering problems, as well as with any thermo(visco)elastic potential

This method exactly preserves the continuum laws of thermodynamics intrinsically while preserving the existing symmetries. The resulting solutions are physically accurate since they preserve the fundamental physical properties of the model. What is more, it shows an excellent performance with respect to the method's robustness and stability. Proof for these claims will be provided in the presentation as well as numerical examples that illustrate its performance.

REFERENCES

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