# High-Order Arbitrary Lagrangian Eulerian One-Step WENO Finite Volume Schemes on Unstructured Meshes for Conservative and Nonconservative Hyperbolic Balance Laws

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## ABSTRACT

In this work we present a new class of high order accurate Arbitrary-Lagrangian-Eulerian (ALE) one-step finite volume schemes for the solution of nonlinear systems of conservative and non-conservative hyperbolic partial differential equations. The numerical algorithm is designed for two and three space dimensions, considering moving unstructured triangular and tetrahedral meshes, respectively.

High order of accuracy in space is obtained by adopting a WENO reconstruction technique, which produces piecewise polynomials of higher degree starting from the known cell averages. Such spatial high order accurate reconstruction is then employed to achieve high order of accuracy also in time using an element-local space-time finite element predictor, which performs a one-step time discretization.

The entire mesh motion procedure is composed by three main steps, namely the Lagrangian step, the rezoning step and the relaxation step. We underline that our scheme is supposed to be an ALE algorithm, where the local mesh velocity can be chosen independently from the local fluid velocity. Once the vertex velocity and thus the new node location has been determined, the old element configuration at time $t^n$ is connected with the new one at time $t^{n+1}$ with straight edges to represent the local mesh motion, in order to maintain algorithmic simplicity.

The final ALE finite volume scheme is based directly on a space-time conservation formulation of the governing system of hyperbolic balance laws. Specifically, the nonlinear system is reformulated more compactly using a space-time divergence operator and is then integrated on a moving space-time control volume. We adopt a linear parametrization of the space-time element boundaries and Gaussian quadrature rules of suitable order of accuracy to compute the integrals.

We apply the new high order ALE finite volume schemes to several hyperbolic systems, namely the multidimensional Euler equations of compressible gas dynamics, the ideal classical and relativistic magneto-hydrodynamics (MHD) equations and the non-conservative seven-equation Baer-Nunziato model of compressible multi-phase flows with stiff relaxation source terms. Numerical convergence studies as well as several classical test problems will be shown to assess the accuracy and the robustness of our schemes.

## REFERENCES

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