Efficient and accurate splitting methods for time integration of multi-physics systems

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ABSTRACT

Coupled multi-physics systems dominate computational models of advanced science and engineering applications. These systems give rise to challenges not typically faced by single-physics models: including interacting stiff and nonstiff processes, disparate time scales, increased nonlinearity and large data requirements. Historically, opposite approaches have been pursued by theoreticians and practitioners for handling such issues. One one side, numerical methods and software have focused on fully-implicit approaches that enable rigorous understanding and control of numerical error, but require optimally scalable nonlinear and linear solvers that may not exist for these complex problems. Meanwhile, practitioners have employed simple operator-splitting techniques that admit optimal solvers for each physical process, but that couple these models in an inaccurate and unstable manner [1-2].

In this work, we investigate mixed implicit-explicit *additive Runge-Kutta* and *generalized additive Runge-Kutta* methods [3-4], as implemented in the newly-available ARKode solver library [5], that attempt to bridge this gap between theory and practice. We investigate a multi-physics problem from computational astrophysics [6], that evolves a tightly-coupled system of radiation transport and chemical ionization to simulate an ionization front in the early universe. These are modeled using a nonlinear system of advection-diffusion-reaction equations. Here, the reaction time scale is significantly faster than both diffusion and advection, although the diffusion must also be treated implicitly for numerical stability. While optimally scalable linear solvers (multigrid) may be applied to the reaction-diffusion system, fully implicit methods suffer in the presence of significant advection. However, through partitioning these processes into an implicit reaction-diffusion component and an explicit advection component, we are able to obtain a scalable and accurate solution to the coupled system.

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