Hybrid-Trefftz Finite Elements for Multiphase Soils

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ABSTRACT

As opposed to the Rayleigh-Ritz method, which forms the basis of all conforming finite element formulations, the alternative Trefftz method searches for approximate solutions of boundary value problems using approximation functions that satisfy locally the (homogeneous) governing differential equation, but do not necessarily meet the boundary conditions, which are enforced by means of collocation, least squares or weighted residuals. Consequently, the functions collected in the approximation basis embody the physical characteristics of the modelled phenomenon, thus endorsing the implementation of macro-elements that exhibit remarkable robustness when confronted with problems that typically hinder the application of conforming finite elements to dynamic problems, such as incompressibility, mesh dependency on the wavelength, gross mesh distortion and modelling of unbounded domains.

This presentation gives a summary of the development of hybrid-Trefftz finite elements for elastodynamic problems defined on multiphase soils, at the University of Lisbon over the course of ten years. The soils are modelled as multi-phase media, formed of a porous solid skeleton saturated by water and (optionally) air. The elements are adequate for both harmonic (frequency domain) and transient (time domain) problems and accommodate both finite and infinite domains.

The harmonic applications are designed to illustrate the basic concepts of the formulation and to assess the convergence patterns under p- and h-refinement, the numerical robustness of the elements, and the effectiveness of the absorbing boundary strategy adopted for modelling unbounded media.

A weak Galerkin approach is used for the discretization in time of transient problems, leading to a series of uncoupled problems in space, which preserve the hyperbolicity of the original problem. The time basis is constructed using Daubechies wavelet functions. For both finite and infinite media, consolidation tests are conducted and the results compared with the predictions of the finite element platform ABAQUS™. Shock simulations are also presented and the results compared with similar analyses reported in the literature.