Anisotropic mesh adaptation for the computation of Lagrangian coherent structures
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ABSTRACT

It is common knowledge that a two-dimensional (2D) steady flow structure, such as a recirculation zone or a stagnation/ separation line, is easily identified with the use of streamlines. However, the analysis of three-dimensional (3D) flows is much more difficult, and pathlines or streaklines often fail to reveal the complexity and interactions of flow structures in a rigorous frame independent setting. Haller's [1, 2] response to this issue stemmed from the theory of dynamic systems applied to the study of particles passively transported by the flow. In this theory, the notion of Lyapunov stability appears, and results in the construction of the finite time Lyapunov exponent (FTLE).

The finite-time Lyapunov exponent (FTLE) is extensively used as a criterion to reveal fluid flow structures, including unsteady separation/attachment surfaces and vortices, in laminar and turbulent flows. However, for large and complex problems, flow structure identification demands computational methodologies that are more accurate and effective.

With this objective in mind, we propose a new set of ordinary differential equations to compute the flow map, along with its first (gradient) and second order (Hessian) spatial derivatives. We show empirically that the gradient of the flow map computed in this way improves the pointwise accuracy of the FTLE field. Furthermore, the Hessian allows for simple interpolation error estimation of the flow map, and the construction of a continuous optimal and multiscale L^p metric. The Lagrangian particles, or nodes, are then iteratively adapted on the flow structures revealed by this metric. It is found that Lagrangian Coherent Structures are best revealed with the minimum number of vertices with the L^1 metric.

This approach will be extended to a novel adaptive method developed in Bois et al. [3], which does not necessitate the computation of the Hessian tensor. This reduces the size of the systems of ODEs to 6 in 2D and to 12 in 3D. We show that very anisotropic meshes can be obtained with similar accuracy to the metric-based approach, but at a lesser computational cost. We also show that the same method can be used for quadratic discretizations of the flow map (and in theory to polynomials of any degree), here again in opposition to metric-based adaptation methods.

REFERENCES

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