Fluid-Porous-Media Interaction:
A Decoupled Solution Algorithm via Localised Lagrange Multipliers

Seyedmohammad Zinatbakhsh* and Wolfgang Ehlers

Institute of Applied Mechanics (CE), Chair of Continuum Mechanics
University of Stuttgart, Pfaffenwaldring 7, 70569 Stuttgart, Germany
Email: Zinatbakhsh@mechbau.uni-stuttgart.de - Web page: http://www.mechbau.de

ABSTRACT

Surface interaction among bulk-fluid bodies and multi-phasic porous media occurs in various circumstances. Some of the well-known examples are the interaction of blood with a blood vessel wall, a body of water with an earth-dam structure, or acoustic waves with acoustic panels used in soundproofing. These are highly coupled phenomena inheriting different coupling mechanisms which take place within the interacting subdomains as well as across the common boundaries. Consequently, the mathematical models of such phenomena also consist of sets of coupled differential equations, which are commonly solved numerically, following a monolithic or a decoupled approach [1]. Here, the focus is on the latter.

To design an algorithm for the decoupled solution of the fluid-porous-media-interaction problem can be advantageous in several ways. First and foremost, the process of decoupling breaks down the problem into several subproblems, which can be solved efficiently employing specialised fluid and porous media solvers. Furthermore, solution of several decoupled subproblems instead of one highly coupled problem may be more economical in terms of computational costs. This is especially the case if the scheme permits parallel treatment of the produced subproblems.

Here, motivated by the idea of the method of localised Lagrange multipliers [2], a partitioned solution algorithm for the problem of surface interaction between an incompressible and inviscid fluid with a saturated biphasic porous medium with intrinsically incompressible and inert constituents is proposed. This method facilitates spatial partitioning of the problem and a parallel solution of the interacting components, and allows for using tailored solvers optimised for each subproblem. Moreover, using the method of perturbed Lagrange multipliers within the subsystems, the pressure fields are eliminated. It reduces the size of the problem, simplifies the formulation of the interface constraints and also removes the burden of using mixed finite elements.

It is known that the decoupled solution of a coupled problem may lead to conditional stability of the produced numerical results [1]. In this regard, the stability behaviour of the resulting staggered approach is analysed, and the unconditional stability of the method is established. To validate this conclusion, the method is employed to solve benchmark examples and, using the numerical results, the reliability of the outcomes of the stability analysis is investigated.

REFERENCES

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