RECURRENCE PLOTS ANALYSIS OF PRESSURE FLUCTUATIONS IN FLUIDIZED BEDS

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Summary. *Recurrence plot (RP) and recurrence quantification analysis (RQA), as powerful statistical techniques, have been used for studying the dynamic behavior of gas-solids fluidized beds. The method of delays was used to reconstruct the state space attractor to carry out analysis in the reconstructed state space. In this work, variance of recurrence rate, which indicates density of recurrence points in RP, against different epoch lengths (time windows) for time series of pressure fluctuation of fluidized bed was calculated. It was concluded that the characteristic parameters of RPs could reflect the extent of chaos in fluidization behavior. The average cycle frequency and entropy as nonlinear dynamical invariants were evaluated with RQA at different aspect ratios. The estimated entropy showed a similar trend of average cycle frequency for different aspect ratios. The results also indicated that the entropy and average cycle frequency are higher in smaller aspect ratios showing that the importance of the finer structures. In addition, a minimum in average cycle frequency and entropy of the pressure fluctuations indicated a minimum deviation from periodicity or, in other words, a minimum deviation from the larger structures, of the bed. The results of this study allow the deep understanding the fluidized bed hydrodynamics which can then be used for scale up.*

1 INTRODUCTION

Fluidized bed reactors have a numerous advantages over other reactor types that make them suitable for industrial applications. They have good particle mixing, high heat and mass transfer rates in addition to low pressure drop. However, due to complexity of the hydrodynamics, design and scaling of this type of chemical reactor are still not straightforward [1-3]. The governing equations of fluidized bed system are rather complex. Since the performance of a fluidized bed is dependent on their hydrodynamic states of fluidization, many investigations reported the hydrodynamic properties of fluidized bed properly such as transition velocities, bubble and cluster characteristics. There are many techniques to determine the hydrodynamic properties of fluidized bed such as optical fiber probes, pressure fluctuations measurements and etc. However, a great advantage of the pressure signals is that they are easy to measure consisting different dynamic phenomena taking place in the bed, such as bubble formation, bubble coalescence and splitting, bubble passage as well as particles behaviors [4].

Traditionally, time series of pressure signals are analyzed using spectral (e.g. Fourier transform, power spectrum) or statistical (e.g. standard deviation, averages) analysis. These analysis techniques assume that the irregular time dependant behavior is due to the linear summing up of random and periodic fluctuations. These techniques do not include the complex hydrodynamic of fluidized beds [1, 5]. Most researchers who investigated fluidized bed based on pressure fluctuations have accepted them as a nonlinear system [5-9]. The new technique that takes account of the nonlinearity of the dynamics is called chaos analysis, in comparison to statistical and spectral analysis [1].

All methods of nonlinear time series analysis are based on the attractor reconstruction of the underlying system in the state space. However, different reconstruction methods may lead to different embedding parameters. In other words, these methods are accompanied by some limitations such as uncertainty on attractor reconstruction methods [9]. Many researchers believe that the two-phase structure of the fluidized bed has a low-dimensional chaotic behavior (typically more than 3 and less than 5) in the state space [9-13]. Thus, attractors with dimensions more than three can be figured only by projection into the two or three-dimensional spaces. On the other hand, long-term data sampling, which is required for typical nonlinear evaluation of the pressure fluctuations in bubbling fluidized bed is usually involved with some difficulties (e.g. steady sate sampling with practical fluctuation feed flow, data saving, data acquisition, etc.) during experimental measurement [12, 14-15].

Recurrence is a basic property of dynamical systems, which can be exploited to describe the system's behavior in phase space [16]. While in the state space, attractors with dimensions more than three cannot be visualized due to constrains in figuring the high dimensional attractors, any phase space trajectory can be represented in a 2-dimensional plot using recurrence plot (RP). In Addition, while embedding is required for reconstruction of attractor in state space, RP may be constructed without embedding. All information contained in the embedded RP can be attained in the non-embedded one [17]. Moreover, the remarkable properties of RP are its ability to evaluate non-stationary and short-term data [18, 19]. These features make RP a very potent tool to study fluidized bed hydrodynamics and eliminates needs for time consuming and difficult long-term data sampling required in typical methods of nonlinear analysis The aim of this work is to apply the RP and recurrence quantification analysis (RQA) to study of scaling aspect ratio (L/D) effect on the dynamic features of the gas-solids fluidized bed using the local pressure fluctuation signals.

2 RECURRENCE QUANTIFICATION ANALYSIS (RQA)

2.1 Recurrence plots

RP technique, derived from nonlinear properties, is based on a graphical explanation of system's dynamics. Ekmann et al. [16] introduced the conception of recurrence plot, as a graphical tool that can determine recurrent behavior in a phase-space of a dynamical system. Briefly, a RP provides a qualitative picture of the correlations between the states of a time series over all available time-scales. A phase-space is generally a high dimensional space and can only be visualized by the projection onto smaller two- or three-dimensional sub-spaces. RPs enable investigation of a m-dimensional phase-space trajectory through a two-dimensional representation of its recurrences to be possible.

Recurrence plot is a 2-dimensional plot expressed by the matrix:

$$R_{i,j} = \Theta(\varepsilon - \left\| x_i - x_j \right\|) \qquad i, j = 1, 2, 3, ..., N \tag{1}$$

where N is the number of measured points, x_i , $x_j \in R^d$ represent the i-th and j-th points of the d-dimensional state space trajectory, $\| \; \|$ represent the norm, ε is a threshold distance and Θ is the Heaviside function. The RP is obtained by plotting the recurrence matrix, Eq. (1), If $R_{i,j} \equiv 1$, it is considered as a recurrence point and appears as a black dot, if $R_{i,j} \equiv 0$, it forms a white dot [18]. March et al. [17] showed that RP can be constructed without embedding. Thus, it was thought desirable to choose the delay time of 1 based on the Takens theorem [20]. Therefore, in the present work, the RP of time series of pressure fluctuations was constructed without embedding.

2.2. Determining Parameters for RQA

The graphical representation of RPs may be complicated to evaluate, since they are considered as qualitative tools to detect hidden rhythms graphically. The quantification analysis of the recurrence plots involves estimation of some parameters (recurrence parameters) that describe the structures in the plots such as single dots and diagonal, vertical and horizontal lines. The structures within a recurrence plot are related to the different dynamics of the system [18]. Recurrence rate and entropy are two of RQA variables that were used in this work.

Recurrence rate (*RR*) expresses the density of repeated states throughout the trajectory and is mathematically defined as:

$$RR = \frac{1}{N^2} \sum_{i,j=1}^{N} R_{i,j} \tag{2}$$

where $\sum R_{i,j}$ is the total number of repeated points. Usually, *RR* is used to determine the value of radius threshold. The radius threshold should not be selected so large that makes the value of *RR* be greater than 20 % [18].

Entropy (*ENT*) refers to the Shannon information entropy of all diagonal line lengths distributed over integer bins in a histogram. Individual histogram bin probabilities ($p(l) = P(l)/N_l$) are computed for each non-zero bin and then summed according to Shannon's equation.

$$Entropy = -\sum_{l_{\min}}^{N} p(l) \log_2 p(l) \tag{3}$$

Entropy is related to complexity of the system. For example, entropy would be expected to be 0.0 bits/bin for a periodic systems in which all diagonal lines are of equal length, but relatively high within chaotic systems [18, 21].

3 EXPERIMENTS

Experiments were carried out in a gas-solid fluidized bed made of a Plexiglas-pipe of 15 cm inner diameter and 2 m height. Air at ambient temperature entered the column through perforated plate distributor with 435 holes of 7 mm triangle pitch. A cyclone was used to separate air from particles at high superficial gas velocities. Sand particles (Geldart B) with mean size of 150 and a particle density of 2640 kg/m^3 were used in the experiments. The bed was operated with different loaded sand heights (*L/D* of 1, 1.5 and 2) and at gas velocities ranging from 0.1 to 1.1 m/s.

Pressure probe (model SEN-3248 (B075), Kobold Company) was screwed onto the gluing studs located 10 cm above the distributor. Pressure fluctuations were recorded in approximately 164 s corresponding to 65535 data. The measured signals were band-pass (hardware) filtered at lower cut-off frequency of 0.1 Hz to remove the bias value of the pressure fluctuations and upper cut-off Nyquist frequency (200 Hz). The sampling frequency was 400 Hz. This sampling frequency is also in according with criterion of 50 to 100 times of the average cycle frequency (typically between 100 to 600 Hz) which is required for nonlinear evaluation of the pressure fluctuations in bubbling fluidized bed [13, 14].

4 RESULTS AND DISCUSSION

In fluidized beds, the main frequency of pressure fluctuations is normally below 10 Hz with a maximum at about 2.5-3 Hz. A difference between the average cycle frequency, f_c (the number of times per time unit the signal crosses its average) and the dominant frequency of the spectrum, f_d, indicates deviations from a perfect periodicity of the macro scale, since f_d is related to macro-scale structures [14]. Figure 1 indicates the average cycle frequency of the pressure fluctuations at aspect ratios of 1, 1.5 and 2. As can be seen, at lower aspect ratio, f_c has the higher deviation from the dominant frequency (2.5-3 Hz) which indicates that the finer structures have significance. In addition, as it can be observed, average cycle frequency of all three different aspect ratios initially decreases and approaches to the peak dominant frequency of the bed and then increases with an increase in velocity.

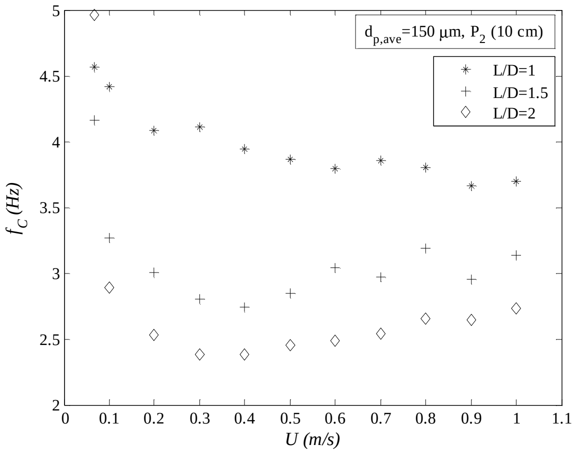
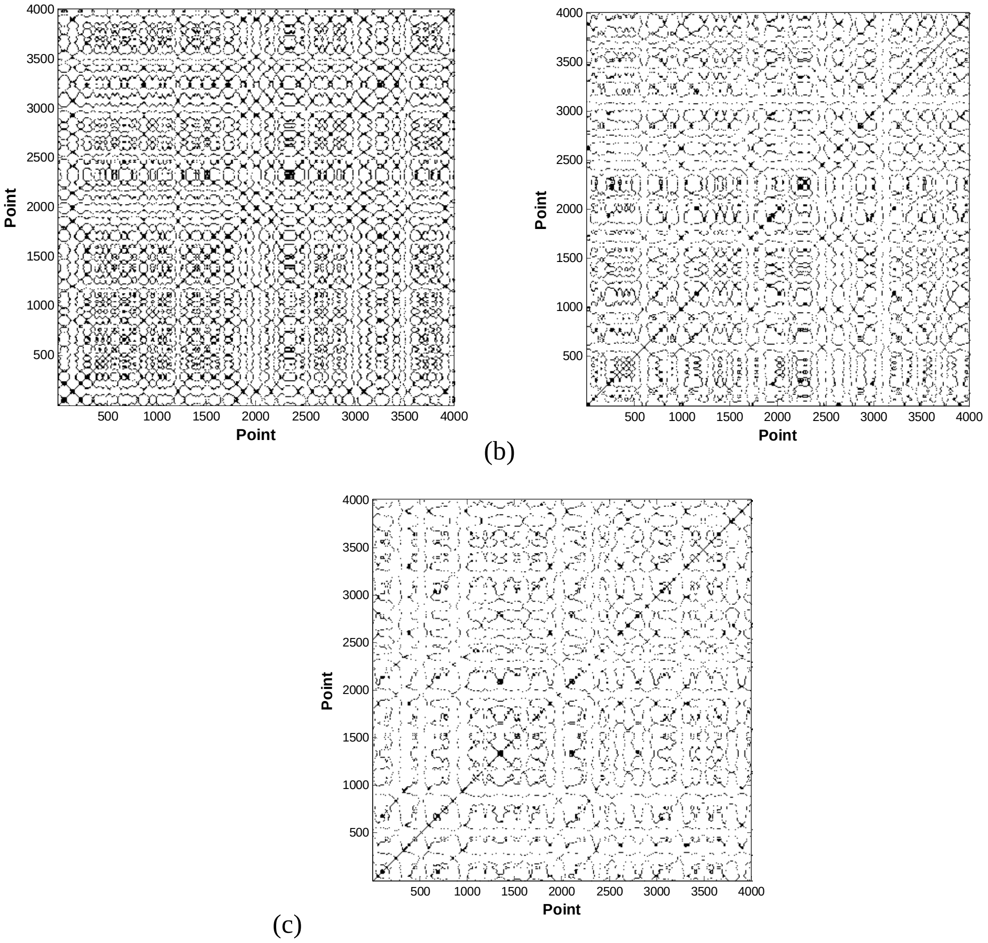


Figure 1: Average cycle frequency (f_c) of the pressure fluctuations at different aspect ratios

Figures 2a-c show the RP constructed from the pressure time series at aspect ratios of 1, 1.5, and 2, respectively. The repeated structures shown in these figures can be categorized in four groups of short diagonal lines, small bold areas, white bands (strips) repeated approximately regular and stretched vertically or horizontally, and quasi-square shapes made of horizontal and vertical lines with white area within them. Each of these typical patterns is linked to a specific behavior of the system [18]. In this paper, only characteristics related to diagonal lines are considered.

The presence of single spots and the large diagonal lines with invariable distance between them is the obvious property of the stochastic and periodic recurrence plot, respectively. Short diagonal lines with irregular distance between them are one of the indications of chaotic dynamics [18, 22]. However, single points are rarely found in the RP of the fluidized bed. In other hands, short diagonal lines in Figures 2a-c show complexity of fluidized bed and indicate that the bed's hydrodynamic behavior is predictable only for short times. This qualitative pattern of RP is further quantified by RQA in terms of *% RR* and *ENT*.



(a) (b) (c)

Figure 2: Recurrence plot of the fluidized bed at aspect ratios of (a) 1 (b) 1.5, and (c) 2; U=0.5 m/s; particles size 150 μm; N=4000; ε=0.1

RQA variables are usually calculated in consecutive epochs and dynamics of the system is inspected through them. Figure 3 shows the plots of the values for each of these quantified parameters (% *RR* and *ENT*) in consecutive epochs. As shown in this figure, the values of *RR* are smaller than 20 % for all aspect ratios. This shows that the value of radius threshold has been chosen reasonably. By comparing of *RR* values of the fluidized bed with the values of the stochastic and periodic systems, it was found that the dynamic behavior of fluidized bed is between long-term predictable and complete unpredictable systems. In addition, *RR* values increased with an increase in aspect ratio. Therefore recurrence rate verifies that the bed with higher aspect ratio shows a more periodic behavior which had been shown by the average cycle frequency results. At this condition, effect of macro phenomena on the pressure fluctuations of the bed are dominant against meso and micro phenomena and the pressure signal approaches periodic behavior. It is expected that a more periodic system has the lower entropy which can be confirmed by the plot of entropy values. As can be seen in this plot, the entropy is smaller in higher aspect ratios. This shows that contribution of the larger structures becomes more important in higher aspect ratios and cause to lower complexity.

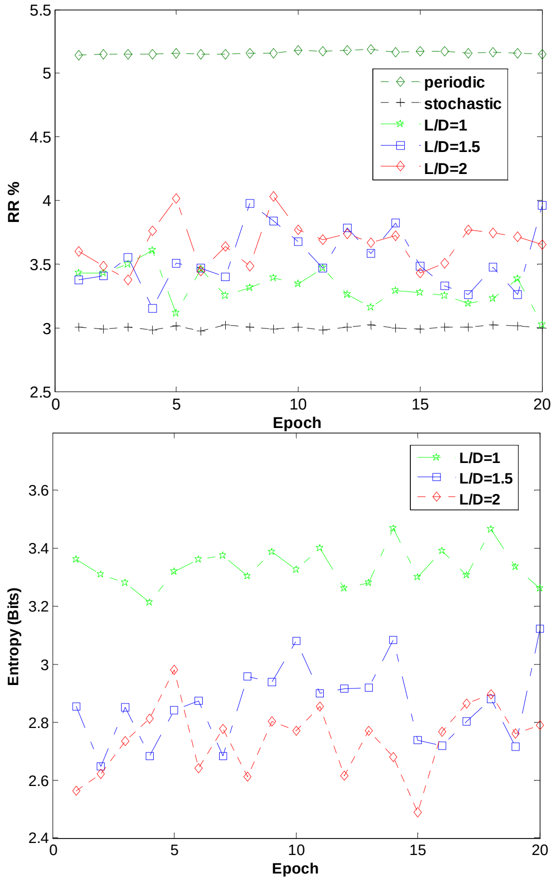


Figure 3: RQA variables (Recurrence Rate and Entropy) of the fluidized bed pressure signal at U=0.5 m/s and different aspect ratios; Epoch length=2000; N=40000; ε=0.05; l_{min}=2.

Figure 4 shows the entropy of the pressure fluctuations measured 10 cm above distributor as a function of gas velocity for different aspect ratios and particles size 150 μm. As shown in this figure, the entropy for all three aspect ratios initially decreases and then increases with an increase in gas velocity. Comparing Figures 1 and 4 reveals that the trends of average cycle frequency and the entropy against gas velocity are approximately similar. It can be concluded that when there is a minimum deviation from periodicity of the bed, entropy are minimum.

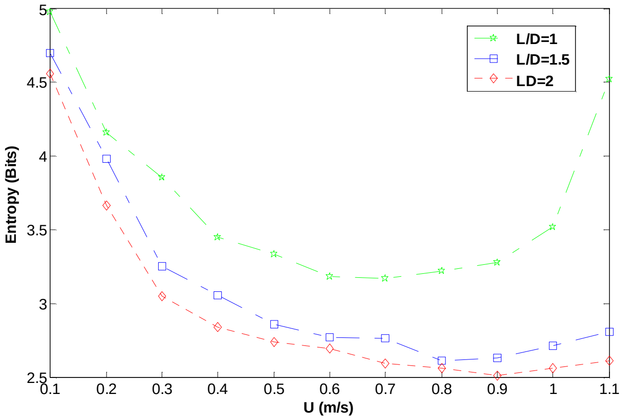


Figure 4: Entropy of the pressure fluctuations at different L/Ds of 1, 1.5 and 2, Epoch length=2000; N=40000; ε=0.05; l_{min}=2.

5 CONCLUSIONS

Recurrence plot (RP) and recurrence quantification analysis (RQA) were used to study of scaling aspect ratio effect on the dynamic features of the gas-solid fluidized bed. The presence of short diagonal lines in the RP showed that the fluidized bed is predictable only for short times. The higher aspect ratio provides greater amplitude due to larger bubbles. This trend is confirmed by the average cycle frequency results too, since at higher aspect ratio, f_c has the lower deviation from the dominant frequency which indicates that the larger structures have importance. At higher aspect ratio, effect of macro phenomena on the pressure of the bed are dominant against meso and micro phenomena and the pressure signal approaches periodic behavior. Recurrence rate and entropy verify this result and show that the bed with higher aspect ratio has a more periodic behavior. The results of the present work showed that the RQA is a powerful and easy method that its variables can be used for scaling, monitoring, study of hydrodynamic behavior within the fluidized bed system.

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