Variationally Relaxed Inelastic Phenomena and the Microsphere Model

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ABSTRACT

A great deal of experimental and theoretical effort has been expended upon carefully characterizing the behaviour of polymeric materials at the chain level. Crowning achievements in this realm include the Gaussian chain model, suitable for long chain polymers stretched to moderate degrees relative to their end-to-end length, and the Langevin chain model for large deformations of chains. These models have been successfully validated experimentally and are thus ubiquitously present in many theories of polymer-system mechanics. Despite these major achievements, it is still challenging to develop mechanical models for networks of polymer chains. The two primary hindrances that give arise to this situation are: (1) chain-chain interactions are difficult to model and (2) these networks are topologically complex and insufficiently characterized. Despite these complexities, the need for a scheme to transition from one-dimensional mechanical response to three-dimensional behaviour is still present.

As an improvement on the well-known affine n-chain models or the more general distributed chain frameworks, Miehe's group proposed the micro-sphere model, which encompasses all of these models by formulating the development of a macroscopic free energy for a network as a constrained relaxation problem -- akin to a classical homogenization problem modulo the gradient constraint. This leads to an elasticity model, similar to Bazant's micro-plane model, but one that is no longer restricted by an affine assumption.

In this presentation we extend the microsphere model by proposing a more general kinematic construction, one that is tensorial in nature and based on the Hencky measure. This permits us to transparently elucidate one of the central assumptions of the original microsphere model regarding the treatment of the volumetric energy. Our proposed model further permits us to incorporate inelastic phenomena such as viscoelasticity, strain induced crystallisation, etc. in a fully non-affine and fully relaxed manner through the exploitation of Biot's Principle.