# Two-scale incremental variational formulation for thermo-mechanical coupled analyses of fiber reinforced thermoplastics

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## ABSTRACT

We present a variational framework of two-scale thermo-mechanical coupled analyses, which can be incorporated with FE2 type multiscale computations for fiber reinforced thermoplastics (FRTP). The incremental variational formulation (IVF) [1] is employed to define the global and local inf-sup problems for thermo-mechanically coupled phenomena within a certain time interval, while two-scale convergence theory in the mathematical homogenization framework is applied to derive micro- and macro-scale incremental potentials. The key ingredient of the formulation is the introduction of equilibrated or non-equilibrated temperature fields, which are related to each other by the time integration factor. It should be noted that the formulation not only has obvious variational structures, but also ensures the thermosdynamical consistency. Also, hyper-dual numbers [2] are utilized to calculate the derivatives of various variables in the finite element equations.

In order to apply our framework to FRTP, we originally formulate a constitutive model of thermoplastic resin based on multi-mechanism theory [3], whereas the fiber of FRTP is modelled by standard thermo-hyperelasticity. The constitutive model consists of rheology elements that represent hyper-viscoelastic and viscoplastic deformations along with kinematic hardening, and is intended to capture not only temperature and deformation rate dependences, but also the transition between glassy and rubbery material responses around glass-transition temperature. When incorporating the constitutive model into the proposed two-scale incremental variational formulation, we assume that viscoelastic and viscoplastic pseudo dissipation potentials can be additively decomposed and that Perzyna's procedure is applicable for viscoplasticity.

Several numerical examples are presented to validate the performance of the proposed constitutive model in representing typical material responses of thermoplastic resins such as stress-softening, strain-recovery and self-heating phenomena. We then perform two-scale thermo-mechanical coupled analyses for FRTP by solving the derived two-scale inf-sup problems to characterize the multiscale mechanical-thermal coupling behaviour of FRTP and discuss the role of the equilibrated or non-equilibrated temperature fields.

## REFERENCES

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