Excluding Fibers under Compression with a New General Invariant in Modeling of Soft Biological Tissues

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ABSTRACT

In general, we can treat each layer of soft biological tissues like an arterial wall as an incompressible fibrous composite which consists of ground substance embedded with three-dimensional (3D) dispersed collagen fibers. Collagen fibers are often organized as a family within which we can define a mean fiber orientation. In modeling of such materials, it is often assumed that collagen fibers do not contribute to the overall mechanical response of the material when loaded under compression. Our group has previously proposed a fiber dispersion model [1] for arterial tissues based on the so-called generalized structural tensor (GST) approach. The contributions of the collagen fibers are incorporated into the strain-energy function through the GST. If the GST is modified to include the fibers under tension only, then it is straightforward to show that this approach may yield negative stress under certain circumstances.

In order to avoid this dilemma and exclude the fibers under compression [2], in this study we introduce a new class of general invariant which only depends on the fibers under tension within a 3D unit sphere. This general invariant is defined as an integration of the fiber strain energies across all the directions under tension. So the fiber strain energies in the directions under compression are automatically excluded. In particular, we provide the expression of strain-energy function based on the general invariant, and the corresponding Cauchy stress and elasticity tensors are presented in a decoupled form. We have implemented the proposed fiber dispersion model in a general purpose finite element program. The incompressibility of the material is enforced by using the augmented Lagrangian method. The performance and implementation of the model are demonstrated by means of uniaxial extension and simple shear tests. The finite element results are in perfect agreement with the analytical solutions obtained by using MATHEMATICA. Further computational studies are necessary to verify this model with more complex boundary and loading conditions.

References

[1] T.C. Gasser, R.W. Ogden and G.A. Holzapfel, Hyperelastic modelling of arterial layers with distributed collagen fibre orientations. *J R Soc Interface* 3:15–35, 2006.

[2] G.A. Holzapfel and R.W. Ogden, On fiber dispersion models: exclusion of compressed fibers and spurious model comparisons. *J Elasticity*, in press.