

FFT-based solver (in finite strain) made simple

T.W.J. de Geus^{*1,4}, J. Vondřejc², J. Zeman³, R.H.J. Peerlings¹, M.G.D. Geers¹

¹ Eindhoven University of Technology, Department of Mechanical Engineering,
Eindhoven, The Netherlands

² Technische Universität Braunschweig, Institute of Scientific Computing,
Braunschweig, Germany

³ Czech Technical University in Prague, Department of Mechanics, Faculty of Civil Engineering,
Prague, Czech Republic

⁴ École polytechnique fédérale de Lausanne, Computational Solid Mechanics Laboratory,
Lausanne Switzerland

* e-mail: tom@geus.me, web page: <http://www.geus.me>

ABSTRACT

Computational micromechanics and homogenization require the solution of the mechanical equilibrium of a periodic cell that comprises a (generally complex) microstructure. Techniques that apply the Fast Fourier Transform, first introduced by [1], have attracted much attention as they outperform other methods in terms of speed and memory footprint. Moreover, the Fast Fourier Transform is a natural companion of pixel-based digital images which often serve as input. In its original form, one of the biggest challenges for the method is the treatment of (geometrically) non-linear problems, partially due to the need for a uniform linear reference problem. In a geometrically linear setting, the problem has recently been treated in a variational form resulting in an unconditionally stable scheme that combines Newton iterations with an iterative linear solver, and therefore exhibits robust and quadratic convergence behaviour [2]. Through this approach, well-known key ingredients were recovered in terms of discretization, numerical quadrature, consistent linearization of the material model, and the iterative solution of the resulting linear system. As a result, the extension to finite strains, using arbitrary constitutive models, is at hand. The resulting framework is straightforward to understand, implement, and possibly extend. In this talk the whole framework will be presented whereby focus will be placed on the many similarities with the Finite Element method. At the same time, emphasis is placed on the simplicity of the implementation. In fact, all presented results are generated using simple, single file, Python scripts; one as short as 59 lines. All of these scripts are available for free download [4]. Finally the power of the method will be illustrated using a statistical analysis of fracture initiation in dual-phase steel [5]. In this study a large batch of micrographs is directly used as input for the numerical analysis.

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