Adaptive Non-linear Model Order Reduction of Computational Homogenisation using a Reduced Strain Basis and Wavelets

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ABSTRACT

Computational Homogenisation is a technique used to accurately obtain the homogenised response emerging from the micro-structure of materials, provided that the principle of separation of scales is respected [1]. The macroscopic behaviour of the material is determined by solving a boundary value problem on representative volume element (RVE) of the micro-structure under macroscopic loading conditions, as shown in Figure 1.

For each macroscopic point, the material behaviour needs to be described by a RVE. The non-linear behaviour of the material requires iterative solution techniques for both the macro- and micro-scale problems. This nested nature of the multi-scale homogenisation makes it computationally expensive to apply the framework to realistic problems with fine discretisations at both scales. Therefore, solving a macroscopic problem using the computationally homogenised behaviour grows quickly in terms of computation time and, in the case of history dependent materials, also in terms of memory-usage.

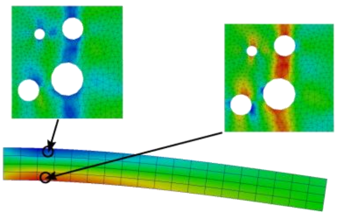

Figure 1: Computational Homogenisation of a cantilever beam

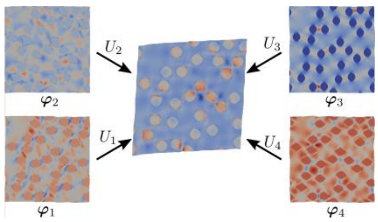

Figure 2: Reduced basis approach to capture the strain field of the RVE

To minimize the computational effort involved in repetitively solving the micro-structure, a reduced order model of the RVE is used. A reduced basis is derived for the strain-field to capture the kinematics, following a classical reduced basis approach [2], see Figure 2. However, it is still costly to integrate the non-linear stress-field, which prevents the effective reduction of the RVE problem at hand. Wavelets have proven to be efficient in reducing computational overhead in the solution of partial differential equations [3]. In this contribution, a novel method combining the reduced basis and an interpolating wavelet basis to efficiently solve the non-linear behaviour of the RVE is presented. The emphasis will be on the accuracy of the reduced model and the computational savings in terms of the number of operations and memory-usage.

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