Selected coupled thermo-mechanical problems solved by discontinuous Galerkin method on polygonal meshes

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ABSTRACT

Discontinuous Galerkin (DG) method [1] is well established computational method for solving boundary value problems. For two-dimensional (2D) problem the DG method can be applied on mesh with triangular as well as with polygonal finite elements [2]. The DG method presented in [2,3] does not require nodes nor shape functions. The approximations are constructed on polynomial basis functions and so very high-ordered approximation can be obtained. In this method the shape of finite elements may be quite arbitrary. i.e. they can have arbitrary number of edges, a hole inside or be non-convex, etc.

In this paper the DG method is applied to non-stationary thermo-mechanical 2D problem. The heat flow through the domain is combined with its mechanical deformations to due mechanical load and thermal expansions. In DG method the integrations through the mesh skeleton have to be performed to enforce continuity of the final solution. In thermo-mechanical problems a special attention have to paid to the skeleton integration. To enforce compatibility the mechanical, thermal and the coupled problems numerical fluxes have to be evaluated on the skeleton. A special technique based on finite difference relations is adopted to set the values of numerical fluxes. Similar approach is applied to enforce boundary conditions of Dirichlet type. Various types of boundary condition are considered which include mixed conditions such as heat convection and displacement-traction conditions. A special postprocessing technique, which is based on Zienkiewicz-Zhu recovery method, is presented.

The method is illustrated with a series of numerical examples in which the correctness and robustness of the method is presented. In one of the example the approximate solution on circular domain is compared with the analytical solution. In other example the mesh with unusual finite elements, e.g. fish-like elements, are used for calculations. Another example deals with stress concentrations on rectangular holes.

REFERENCES

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