# A Numerical Framework for Self-Similar Problems in Plasticity: Indentation in Single Crystals

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## ABSTRACT

A new numerical framework specialized for analyzing self-similar problems in plasticity is developed. Self-similarity in plasticity is encountered in a number of different problems such as stationary cracks, void growth, indentation etc. To date, such problems are handled by traditional Lagrangian procedures that may be associated with severe numerical difficulties relating to sufficient discretization, moving contact points, etc. In the present work, self-similarity is exploited to construct the numerical framework that offers a simple and efficient method to handle self-similar problems in history dependent materials. The procedure allows for focusing the mesh only in regions of interest giving highly detailed results in fractions of the time compared to traditional frameworks. The framework is not limited to a specific constitutive law and may be applied to a wide range of material models. The technique is here applied to wedge indentation in elastic-viscoplastic single crystals.

The three most common metal structures are investigated, namely the FCC, BCC, and HCP crystal structures, where the slip rate fields and stress fields will be compared to analytical predictions [1][2] and traditional numerical simulations [3] when possible. To mimic the condition for the analytical predictions, the wedge indenter is considered nearly flat and the material is perfectly plastic with a very low yield strain. Under these conditions, [1][2] proved analytically the existence of discontinuities in the slip rate field. The numerical simulations reveal a striking match to the analytical prediction showing the expected discontinuities in the slip rate field. In addition, the current results provide much more detailed views of the stress and slip rate fields than previously obtained. The results are all obtained without encountering many of the issues related to the traditional procedures and guarantees that it is indeed the self-similar solution that has been found.

## REFERENCES

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