Parallel 3D Limit Analysis via the Alternating Direction Method of Multipliers

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ABSTRACT

In civil engineering is often necessary to predict the maximum load that can be supported by a structure, known as collapse load. For the analysis of mechanical structures subject to noncyclic loading and presenting a perfect plastic behaviour, the limit analysis theorems prove to give good results and to be an efficient and competitive strategy comparing with other methods (*e.g.* elastoplastic incremental analysis). The static and kinematical limit analysis theorems allow the computation of lower and upper bounds for the collapse loads of these mechanical structures.

Nowadays, and in spite of the remarkable evolution of computers performance, the determination of accurate collapse load estimates can still represent a significant computational effort. In fact, when strict high quality bounds solutions are searched, it is required a high degree refinement of the mesh, leading to substantial CPU and RAM needs, particularly for 3D problems.

In the current work, two complementary finite element limit analysis formulations are presented, namely, an upper and a lower bound formulation. These models lead to optimization problems with a significant number of decision variables and constraints.

The alternating direction method of multipliers (ADMM) technique has been used by the authors to solve these large-scale nonlinear convex optimization problems, because it is a very versatile (only need to ensure that the functions must be closed, proper and convex) and robust (practically never fails to converge) technique and is also inherently parallelizable.

The ADMM has an iterative solution scheme, based on an operator splitting algorithm, that is not only easy to implement but also suitable to efficiently solve large-scale variational problems with parallel processing [1]. In this approach, the optimization problem is divided in two subproblems. The first results in a quadratic unconstrained problem, easily solvable through a linear system of equations. Conveniently, the governing system matrix remains unaltered during the entire iterative process. In fact, all the modifications affect only the right hand side term. Therefore, a single matrix factorization procedure is needed throughout the whole iterative process. The second subproblem collects all the nonlinear part of the original problem, *i.e.*, consists in a sum of small independent optimization problems. In most cases, using nonsmooth techniques to deal with not everywhere differentiable function, it is possible to derived closed-form solutions, otherwise special-purposed algorithms can be applied efficiently. In summary, the first subproblem takes into account the contribution of all finite elements simultaneously, while the second subproblem is carried out at element level, meaning that it is solved independently for each element [2].

In order to illustrate the efficiency and competitiveness of this strategy some numerical examples, related to geotechnical problems, are shown.

REFERENCES

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