A Numerical Method for Calculating the Strain Rate Intensity Factor

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ABSTRACT

Using the classical model of rigid perfectly plastic solids the strain rate intensity factor has been introduced in [1] as the coefficient of the leading singular term in a series expansion of the equivalent strain rate in the vicinity of maximum friction surfaces. In particular, the equivalent strain rate tends to infinity as the normal distance to the máximum friction surface tends to zero. The máximum friction surface is defined by the condition that the friction stress at sliding is equal to the shear yield stress. In the case of hyperbolic systems of equations, an alternative formulation is that the friction surface coincides with an envelope of characteristics. The strain rate intensity factor controls the magnitude of the equivalent strain rate in a narrow layer near maximum friction surfaces. On the other hand, the equivalent strain rate is involved in evolution equations for parameters that characterize the microstructure of material. Therefore, the strain rate intensity factor controls the evolution of material properties near surfaces with high friction. Experiment confirms that the change in material properties in a narrow layer near friction surfaces is much more significant than in the bulk. Therefore, it is reasonable to hypothesize that a material model based on the strain rate intensity factor may adequately describe such a high gradient in material properties. The development of such a model requires an efficient numerical approach for calculating the strain rate intensity factor. The present paper deals with such an approach based on the method of characteristics. The approach is applicable for plane strain problems in plasticity.

The general expression for the strain rate intensity factor found in [1] is rewritten in characteristic coordinates. It is shown that the equivalent strain rate is inversely proportional to the radius of curvature of one of families of characteristics in the vicinity of an envelope of characteristics. It is evident that this radius of curvature vanishes at the friction surface. The coefficient of the leading singular term is written in terms of velocities and space derivatives of velocities in characteristic coordinates. Therefore, this coefficient can be calculated with a very high accuracy using the traditional slip line method. Thus the strain rate intensity factor is also found with a very high accuracy.

As an example, compression of a thin plastic strip between two rough, parallel plates is considered. The method proposed in [2] is adopted to find the network of characteristics and the velocities referred to the characteristic coordinates. It is shown that the strain rate intensity factor is given by a discontinuous function. There are two sources of this discontinuity. One source is a discontinuity in radii of curvature of characteristics in the vicinity of the free surface of the strip. The other source is a discontinuity in velocity at the rigid plastic boundary. The accuracy of the numerical solution is confirmed by comparing with an analytic solution.

REFERENCES

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