

Minimizers in Multi-Physics of Solids. Application to Phase Field Modeling of Fracture

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ABSTRACT

The lecture investigates the construction of new minimization principles for multi-field scenarios in solids and their exploitation with regard to a stable finite element design and the analysis of the material and structural instabilities of the coupled problems. Typical applications cover pure mechanical multi-field problems such as recently developed phase field approaches to brittle and ductile fracture, as well as their multi-physics extensions to heat and mass transport or electro-magnetic coupling in solids. A prototype example is the analysis of hydraulic fracture in poro-hydro-mechanics that governs so-called fracking processes.

Multi-field boundary-value problems are often formulated by choosing on the non-mechanical side the most convenient primary variables, which are often scalar in nature. Typical examples are the chemical potential in diffusion, the temperature in heat conduction, the electric or magnetic potentials in smart functional materials. However, rate-type and incremental potentials defined in terms of these variables typically result in saddle point principles, which govern the evolution of these coupled problems. These structures are convenient on the first sight, but have critical drawbacks: First, they cannot be used for energetic definitions of structural and material stability of the coupled problems. Second, associated mixed finite element implementations must satisfy the LBB condition to guarantee numerical stability.

In this lecture, we propose a general concept for the construction of minimizers in coupled problems, which is based on generalized energy functions. The minimization principles obtained are formulated in a non-standard fashion by using the 'most fundamental combination' of primary variables in combination with certain constraints on the evolution problem, which recast the boundary-value problems into a non-standard format. In this spirit, typical primary variables on the non-mechanical side are the entropy, mass and heat fluxes as well as electric or magnetic inductions. The formulations obtained may be considered as the 'canonical settings' of the multi-field problems under consideration. They overcome the above mentioned problems, by allowing the most elementary definitions of stability and an unconstrained finite element design in appropriate function spaces. Furthermore, they serve as a fundamental basis for the definition of homogenization and relaxation methods in multi-scale analyses of coupled problems.

We demonstrate these features for the model problem of a recently developed phase field approach to brittle and ductile fracture in poro-hydro-mechanics. This application includes an accompanying analysis of instabilities in complex environments related to hydraulic fracturing scenarios.

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