Efficient mixed tetrahedral element for simulation of SMA structures

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ABSTRACT

Medical NiTi stents are structures with complex geometry which exhibit strong material nonlinearity and undergo significant configuration changes. The focus of the present paper is to develop a finite element scheme devoted to the numerical simulation of such class of structures.

When dealing with complex geometries, the use of tetrahedral elements is almost unavoidable. Generally, adopting tetrahedrons instead of hexahedrons, increases the number of elements for a fixed mesh size. In order to limit the relative computational cost, the introduction of corner rotational degrees of freedom, in addition to standard translational ones, has been recognized as an attractive strategy to pursue intermediate accuracy between linear and quadratic elements with translations only, while still keeping a low computational cost compared to the standard formulation.

In the present work, a new mixed tetrahedral element, particularly indicated for problems exhibiting strong material and geometric nonlinearities, is proposed. Since many nonlinear constitutive models are expressed in direct stress-strain form and multiple deformation states may be associated to the same stress level, the element derivation is based on a Hu-Washizu type functional, modified to include independently interpolated rotations. Consequently, skew symmetric stresses are introduced to stabilize spurious zero-energy deformation modes. The element is equipped with translational and rotational corner DOFs, producing a continuous quadratic interpolation of the displacement field and a first-order lagrangian interpolation for the rotation field. Symmetric stresses and strains are equally approximated by means of linear shape functions, suitably constrained to yield a frame invariant and isostatic element. Stress and strain interpolation parameters are condensed out at element level. Extension to geometric nonlinear problems is pursued by the polar decomposition based co-rotation approach proposed in [2], achieving desirable properties as versatility, ability to return a body-attached frame for rigid bodies, invariance with respect to element node ordering and finite-stretch patch test satisfaction. The constitutive state update algorithm for 3D phenomenological models for shape memory alloys (SMA) is based on a non-smooth convex incremental energy minimization concept [3,4]. The method is robust inasmuch it permits a complete detection of all singularities relevant to the energy formulation of the constitutive model, without introducing any regularization in the model and addresses an innovative algorithm for the computation of the dissipation functional in terms of Haigh-Westergaard invariants, allowing for a more general choice of the transformation function. Numerical simulations are presented to validate the proposed mixed tetrahedron and to compare its performances with other available finite element formulations.

REFERENCES

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