Hierarchical finite-strain phase-field model for twinning and martensitic transformation

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ABSTRACT

A hierarchical finite-strain model is developed for phase-field modelling of twinning and martensitic transformation in shape memory alloys. The model is set in the energy minimization framework so that the constitutive equations are fully defined by specifying the free energy function and the dissipation potential. The free energy involves the bulk and the interfacial energy contributions, the latter describing the energy of diffuse interfaces, which is typical for phase-field approaches [1].

In comparison with other finite-strain phase-field models for twinning in shape memory alloys [2,3,4], the present model uses another mixing rule within diffuse interfaces which can be directly generalized to multiple variants or phases with volume changes treated in a consistent manner. Furthermore, the inequality constraints are here directly imposed on the order parameter interpreted as the volume fraction so that it does not violate the physically motivated bounds. In the numerical implementation, the augmented Lagrangian method is used for that purpose [5].

Modelling of microstructures that involve austenite and twinned martensite requires further extension of the model. Two order-parameters are introduced to adequately describe the three involved constituents: the austenite and two variants of martensite. The order parameters are arranged hierarchically so that one parameter differentiates between austenite and martensite while the second one differentiates between the two variants of martensite. As a result, two types of interfaces (austenite-martensite and martensite-martensite) are described in a natural manner. The bulk free energy and the dissipation potential are generalized accordingly.

The model has been implemented in the finite element method and has been applied to study size effects in a CuAlNi shape memory alloy undergoing the cubic-to-orthorhombic transformation. The results of phase-field modelling are compared to the results of sharp-interface models developed in [6,7].

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