Dimensional hyperreduction of general, nonlinear finite element models

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ABSTRACT

In this work, we propose and describe a rigorous framework for the dimensionality reduction of general, parametrized nonlinear finite element models. The first step in the approach consists in the standard Galerkin projection of the semi-discrete motion equations onto a reduced-order space. The basis for this reduced-order space is determined by solving the FE equations for appropriately selected values of the input parameters (for instance, boundary conditions or material parameters). The resulting set of displacement solutions is then processed using a data compression algorithm (in this case, we have used the Singular Value Decomposition, abbreviated SVD), in order to identify and unveil the dominant displacement modes of the problem, which will constitute the desired reduced basis.

The second step of the approach is less standard, and concerns the reduction in complexity of the vector of internal forces arising from the previously outlined Galerkin projection. Although the number of entries of this vector has been already reduced, its complexity still depends on the size of the finite element mesh —evaluating such a vector at each time step and iteration requires, in general, the solution of nonlinear constitutive equations for stresses and internal variables at all quadrature points. The key for reducing the complexity of this vector lies in the decomposition of this internal force vector into the product of a parameter-independent matrix of strain modes, on the one hand, and a parameter-dependent vector of global stresses, on the other hand. The reduction in complexity is achieved by replacing this global stress vector by a low-dimensional interpolant. To construct the basis matrix for this interpolant, we solve the projected motion equations for representative input parameters, and then process the resulting stress solutions, using again data compressing algorithms such as the SVD (as in the first step), in order to uncover the dominant stress modes of the problem. The basis matrix for the interpolation, however, cannot be solely formed by these dominant stress modes, for it gives rise to ill-posed problems. We demonstrate that, to amend this shortcoming, one has to include also the reduced strain modes obtained in the decomposition of the internal forces. This approach is a generalization for general dynamic problems of the "expanded basis" strategy advocated by the authors in Ref. [1] to address the particular case of the self-equilibrium problems appearing in standard computational homogenization of heterogeneous materials.

The efficiency of the proposed approach is assessed in the solution of several structural problems. Results obtained show that the proposed strategy gives rise to well-posed reduced-order problems, with gains in performance with respect to full-order analyses of above two orders of magnitude, and approximation errors below 10%.

REFERENCES

[1] JA Hernández, J Oliver, AE Huespe, MA Caicedo, and JC Cante. High-performance model reduction techniques in computational multiscale homogenization. Computer Methods in Applied Mechanics and Engineering, 276:149–189, 2014.