

# Size-scale Effects in Homogenization of Long Fiber Reinforced Composites

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## ABSTRACT

Fiber micro-buckling represents an important failure mode for long fiber reinforced composite materials. This work discusses a homogenization technique, that accounts for the fiber bending stiffness, so that the numerically homogenized material exhibits a realistic behavior also at size-scales close to the fiber diameter.

The proposed model is inspired to large extent by the model of Fleck & Shu, [1], that accounts for the fiber bending stiffness as well. However, instead of considering a yield criterion for the composite material per se, our model relies on two independent material laws for the fiber and the matrix materials, similar to the size-scale independent model of Christoffersen & Jensen, [2]. Additionally, plastic deformation is expressed in the framework of hyper-elastic instead of hypo-elastic plasticity.

More specifically, the proposed homogenization technique relies on internal kinematic variables, that express the deviation between the deformation gradients in the matrix and the reinforcing fibers. Additional equations for specifying these internal kinematic variables are provided by the force equilibrium between the matrix and the fibers. Proper treatment of the micro-mechanical kinematics of the composite material results to a final formulation which includes spatial gradients of the introduced internal kinematic variables, giving rise to the aforementioned size-scale dependence in the homogenized material behavior. The finite element method is employed for the discretization of the final set of equations corresponding to the macroscopic and micro-mechanical force equilibria as well as the plastic yield and hardening in the matrix material.

In order to evaluate the accuracy of the proposed homogenization, comparisons are provided for numerical examples calculated both with an explicit discretization of the composite micro-structure as well as with the homogenized model. Although the main aim of this work is a contribution to the study of micro-buckling in long fiber reinforced materials, at the same time it provides some insight into the micro-mechanical nature of higher order terms in material laws.

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## REFERENCES

- [1] N.A. Fleck and J.Y. Shu, "Microbuckle initiation in fibre composites: A finite element study", *J. Mech. Phys. Solids*, **43**(12), 1887-1918 (1995).
- [2] J. Christoffersen and H.M. Jensen, "Kink band analysis accounting for the microstructure of fiber reinforced materials". *Mech. Mater.* **24**(4), 305-315 (1996).