A consistent homogenization theory and the numerical implementation of a higher order plasticity model from meso to macro

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ABSTRACT

Classical plasticity models, being scale independent, cannot capture any size depedent behavior. A remedy is to adopt a gradient plasticity model at the sub-granular (meso) scale to account for the different micro-processes leading to the various size effects. In this contribution, we adopt, at the meso scale, the isotropic gradient plasticity model in [1] which incorporated the plasticity rotation effect, to mimic the behavior of an analogous crystal plasticity model with multiple slip systems [2].

For a generic problem, the interaction and competition between the different processes are captured through three length scale parameters [3]: a microstructural length scale characterizing the intra-granular fluctuations, the grain size describing the grain boundary effect, and a structural length scale accounting for the structural effect. Such a high resolution approach, however, is computationally expensive for a large problem since the FE discretization has to be done at a sub-granular level.

This motivates a novel homogenization theory in [4] that translates the isotropic plasticity model from meso to macro. First, we introduce an additional kinematic field to characterize the average surface deformation of a unit grain. Next the Hill-Mandel condition is applied to extract the homogenized governing equations at the macro scale. Departing from most homogenization approaches, we furthermore impose the equivalence of energy and dissipation across the two scales to determine the macroscopic constitutive relations and plasticity flow rules. The scale translation framework is thus thermodynamically consistent, with the three length scale parameters manifesting themselves in the homogenized solutions. This allows for a direct study on the interaction between the different micro-processes, with a lower computational cost compared to a detailed crystal plasticity model.

For an idealized bending problem, we illustrate in [4] the excellent match between the homogenized solutions and meso solutions for the two limiting cases – microfree and microhard conditions at the grain boundaries. It is also highlighted that through the homogenization framework, the strain gradient plasticity model at the meso scale translates into a *micromorphic* continuum at the macro scale.

The homogenized plasticity model is next implemented numerically, and its predictive capabilities furthermore demonstrated through a plane indentation problem. For the aforementioned two limiting cases, the homogenized solutions resemble closely the reference meso solutions, both in terms of the load displacement graphs and the plastic strain profiles. It is noteworthy that the excellent predictions are obtained with a significantly lower computational cost.

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