An incremental energy minimization state update algorithm for isotropic elastoplastic materials

N. A. Nodargi, P. Bisegna

Department of Civil Engineering and Computer Science
University of Rome "Tor Vergata", Via del Politecnico 1, 00133 Rome, Italy
e-mail: nodargi@ing.uniroma2.it, web page: http://www.dicii.uniroma2.it

ABSTRACT

The solution of elastoplastic structural problems in a typical finite element implementation requires the numerical integration of the constitutive equations at each Gauss point for every equilibrium iteration. In the framework of rate-independent small-strain elastoplasticity, such problem is usually addressed by elastic predictor, plastic corrector return mapping algorithms. More specifically, in case the trial stress, i.e. the stress corresponding to no plastic evolution, is not plastically admissible, the plastic evolution equations are solved with respect to the updated stress (e.g., see [1]).

However, the yield functions adopted to model a wide variety of media, such as granular and geological materials, ceramics or high-strength and shape memory alloys, may exhibit features which prevent the use of standard return mapping algorithms (for instance, see [2]). Examples are yield functions which admit false elastic domains, are not defined in the whole stress space or imply high-curvature points of the yield domain (e.g., see [3]).

In the present work, an associative flow rule is adopted in order to avoid the pathological issues descending from the intrinsic ambiguity of flow potential definition [2]. Accordingly, the elastoplastic evolution equations are set in the framework of generalized standard materials and the plastic evolution is shown to follow the minimizing path of a suitable incremental energy, given by the sum of free energy and of dissipation potential, with respect to the increment of plastic strain (for instance, see [4, 5]). Under the assumption of isotropic material behaviour, implying coaxiality of trial stress, increment of plastic strain and updated stress, the unknowns are reduced from six to three. Then, resorting to the cylindrical tensor basis associated to Haigh-Westergaard coordinates (e.g. see [6]), the problem is recast as two nested nonlinear scalar equations, thus obtaining a globally convergent state update algorithm even for yield functions with difficult features. Simple algebraic operations allow the tensorial reconstruction of the stress and of the increment of plastic strain. In addition, a simple representation of the consistent elastoplastic tangent is derived. It is worth pointing out that no tensor inversion is required either in the return mapping iteration or in the computation of the consistent material tangent.

The proposed state update algorithm is validated by comparison with benchmark semi-analytic solution. Numerical results on single material points and finite element simulations are reported for assessing its accuracy, robustness and efficiency.

REFERENCES

[1] J.C. Simo and T.J.R. Hughes, *Computational inelasticity.*, Springer, New York, (1999).

[2] R.M. Brannon, and S. Leelavanichkul., "A multi-stage return algorithm for solving the classical damage component of constitutive models for rocks, ceramics, and other rock-like media.", *Int. J. Fract.*, **163**, 133-149 (2012).

[3] S. Stupkiewicz, R. Denzer, A. Piccolroaz and D. Bigoni, "Implicit yield function formulation for granular and rock-like materials.", *Comp. Mech.*, **54**, 1163-1173 (2014).

[4] W. Han and B.D. Reddy, *Plasticity. Mathematical theory and numerical analysis*, Springer Verlag, Berlin, (1999).

[5] C. Miehe, J. Schotte and M. Lambrecht, "Homogenization of inelastic solid materials at finite strains based on incremental minimization principles. Application to texture analysis of polycrystals", *J. Mech. Phys. Solids*, **50**, 2123-2167 (2002).

[6] M.X. Chen, Q. Peng and J. Huang, "On the representation and implicit integration of general isotropic elastoplasticity based on a set of mutually orthogonal unit basis tensors.", *Int. J. Numer. Methods Eng.*, **99**, 654-681 (2014).