A mixed finite element formulation for geotechnical problems involving localization in plasticity

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ABSTRACT

Fracture can be defined as the onset of a displacement discontinuity in a solid. In the catastrophic failure of slopes, the creation of a slip line in the soil triggers the release of a mass that may develop into a landslide or a rockslide. The determination of the initial detached volume is essential in the stability analysis for safety assessments.

From a computational point of view, this problem poses numerous challenges to be overcome. Within the framework of standard irreducible finite elements, the smeared crack approach [1] allows treating discontinuity as a band of finite width, where the displacements are continuous and the strains are discontinuous, but bounded. Nevertheless, this hypothesis is well-known to present serious numerical drawbacks. Solving problems that involve strain softening, spurious mesh dependence appears and the fracture line direction is biased. Moreover, when isochoric behaviour is enforced (as in the case of undrained soil), locking of the stresses provokes pressure oscillations, with the consequent pollution of numerical calculations. Both problems can be shown not to be related to the mathematical statement of the continuous problem but, instead, to its discrete (FEM) counterpart.

Recently, the authors proved that strain localization numerical issues can be easily alleviated using a Mixed Finite Element formulation, in terms of strain and displacements [2]. As it was reported by Badia and Codina [3] and, then, by Cervera et al. [4], the order of convergence of strains (and stresses) in mixed formulations is one order higher than the displacement-based method, even in case of localized discontinuities. The strain-displacement mixed finite element formulation provides enhanced stress accuracy for a given mesh and allows the determination of localization bands without the introduction of auxiliary tracking techniques. Examples of compressible and incompressible plasticity were presented in [5], where, in particular, it was shown that the energy dissipation is exactly matched in the case J2 plasticity, confirming the consistency of the method.

From these premises, various problems of geotechnical interest are tackled to demonstrate the capabilities of the formulation. The standard irreducible formulation is compared with the introduced mixed formulation in problems involving stability of slopes and shallow foundations.

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