A Finite Element basis for Distortion Gradient Plasticity theory

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ABSTRACT

Experiments and dislocation dynamics simulations have shown that metallic materials display strong size effects at the micron and sub-micron scales, with smaller being stronger. Such size effects have been attributed to geometrically necessary dislocations associated with inhomogeneous plastic flow and much research has been devoted to modelling observed size effects in the past two decades. Most attempts to model size effects in metals have been based on higher order continuum modelling and several Strain Gradient Plasticity (SGP) theories have been developed in order to incorporate some length-scale parameters in the constitutive equations.

Gurtin [1], and more recently Bardella [2], highlighted the relevance of including the plastic spin (i.e., the skew-symmetric part of the plastic distortion) in the higher-order phenomenological modelling of small-scale plasticity, even under small strains and rotations, in what here we call Distortion Gradient Plasticity (DGP). However, despite the superior modelling capability of DGP with respect to SGP (see [2]), the literature is scarce about the development of a general purpose finite element framework for DGP theory.

The aim of this work is to develop a novel finite element framework for the higher order phenomenological modelling of DGP. Both energetic (recoverable) and dissipative (unrecoverable) distortion gradient dependencies will be considered, as in the generalized theory proposed in Section 12 of [1]. The numerical formulation is developed by following and extending the mathematical principles established by Fleck and Willis [3] for SGP. Several boundary value problems are solved and thoroughly analysed to shed light on the role of the plastic spin. Results show that the material parameter governing the dissipation due to the plastic spin strongly affects both the quality and the magnitude of the size effects which can be described by DGP. Various forms of the free energy potential, characterising the energetic higher order terms involved in the theory, are also thoroughly investigated.

REFERENCES

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