A Variationally Consistent Thermoplastic Framework for Microstructure Evolution of Metals Undergoing Phase Changes

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ABSTRACT

The paper deals with numerical modelling of the thermoplastic response of metals undergoing phase changes. Contrary to existing solutions [1, 2], the proposed approach is based on minimizing an incrementally defined potential. The derivation of such a potential, albeit in the somewhat general context, was described for the first time in [3]. Starting from this general model, an extension to non-associated thermoplasticity suitable for cyclic plasticity was presented in [4],

In this paper, the ideas proposed in [4] are further developed and applied to problems involving phase changes. The model is set in a finite strain setting and captures microstructure evolution due to phase changes from the liquid to the solid state. The underlying idea for the description of phase changes is based on [2]. Starting from the balance equations of continuum mechanics, accompanying boundary conditions and the constitutive model, a suitable rate potential is defined first. Subsequently, a time discretization of this potential is carried out resulting in an incremental potential. This potential is the foundation of the presented numerical scheme. To be more explicit, the minimizer of this potential defines the complete state of the considered continuum – including the evolution of microstructures. The predictive capabilities of the final model are demonstrated by means of illustrative numerical examples.

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