Recursive calculation of consistent tangent operator in elasto-plasticity

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ABSTRACT

A nonlinear boundary value problem in continuum solid mechanics is, nowadays, efficiently solved with the finite element method, and an appropriate integration scheme is adopted to numerically tackle the given material constitutive behaviour. With reference to the latter, a wide class of return-mapping algorithms based on an operator-splitting methodology prevails, particularly, in the computational rate-independent plasticity.

Basically, the operator-splitting methodology follows the additive split of the constitutive equations into the elastic predictor part and, when required, the subsequent plastic corrector part with the elastically predicted state variables iteratively projected back onto the yield surface. Such methods, usually also referred to as elastic predictor-plastic corrector algorithms, are classically performed according to the backward-Euler approach. Because of the chosen backward-Euler approach, an implicit system of equations is obtained, which is classically solved iteratively with the Newton-Raphson algorithm. Alternatively, if the operator splitting methodology is performed according to forward-Euler approach, the corrector part direction is evaluated according to the current state. Such approach results in an explicit system of equations which is solved without matrix inversion. Such methods are usually referred as the cutting-plane algorithms and are, according to their explicitness, simpler to implement into a FEM code and computationally more efficient. If summarizing in mathematically more general way, when newly developed constitutive model is implemented into finite element method program, a time integration scheme for solving of model's equations must be chosen. This gives the system of equations which is commonly solved iteratively with the Newton-Raphson algorithm. Finally, it can be concluded the solution strategy roughly remains unchanged regardless the form of constitutive model.

Although in principle a general solution method is available, the last challenge due to implicit FEM remains – the derivation of consistent tangent operator (CTO). The consistent tangent operator is needed to ensure the quadratic convergence of global Newton-Raphson algorithm which governs the fulfilment of equilibrium equations. Classically the derivation of consistent tangent operator is model dependent and when developing new constitutive model special attention must be addressed.

In this contribution the recursive approach to calculation of the consistent tangent operator is presented, where the iterative solution procedure (such as Newton-Raphson algorithm) is applied to integrate the constitutive equations. Thus the consistent tangent operator becomes a part of iterative solution procedure without additional care needed for its derivation. The derivation is based on variational analysis of algorithmic equations with respect to the strain dependency of state variables in each iteration. Consequently, the proposed approach results in the analytically derived recursive formulation of the CTO for a general class of plasticity models. An extension to other constitutive models, e.g. viscoplasticity, represents no additional effort.

The approach is applied on a case, where the integration of constitutive equations is done iteratively with the cutting-plane algorithm (CPA) [1]. With the proposed approach the CTO for CPA was derived for the first time.

REFERENCES

[1] B. Starman, M. Halilovič, M. Vrh, B. Štok, *Consistent tangent operator for cutting-plane algorithm of elasto-plasticity*. Comput. Methods Appl. Mech. Engrg, **272,** 214-232 (2014).